Emory University
Practice Final Exam
MA-210 Advanced Data Science Calculus
Date: May 1, 2025
Instructor: Head TA Mitchell Scott

Student ID _____ Name: _____

Please read the following instructions carefully.

- ***This is simpliy a review session!*** This question booklet contains 6 questions, 2 pages (including the cover) for the total of 95 points/marks. Check to see if any pages are missing. DO NOT scribble or do rough work or make any stray marks on it. Use separate sheet for rough work.
- This is meant to represent what an actual exam might look like. **Read the instructions for individual questions carefully** before answering the questions.
- No instructor was consulted for the making of this assignment.

Question	Points	Score
1	5	
2	10	
3	10	
4	28	
5	22	
6	20	
Total:	95	

1. (5 points) **Exponential variables can also be used to model situations where certain events occur with a constant probability per unit length, such as the distance between mutations on a DNA strand, or between roadkills on a given road. Let X be a random variable pulled from this exponential distribution, where the pdf is**

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0, & \textbf{elsewhere}. \end{cases}$$

What is the $\mathbb{E}[X]$? (n.b. $\lambda > 0$ is what is called the rate parameter, but you can think of it as a constant.)

2. (10 points) **Find the volume of the solid that lies below the function surface given by $f(x, y) = 6x^2y + 20x$ and lies above the region in the xy-plane bounded by $y = x^2$ and $y = 2 - x^2$.**

3. (10 points) **Let $f(x, y) = 4x^2 + 3y^2$. Suppose x and y are functions of t such that**

$$x(t) = \sin t$$
$$y(t) = \cos t$$

Compute $\frac{\mathrm{d}f}{\mathrm{d}t}$ at $t = 3\pi/4$.

4. (14 points) **President Fenvez has a paperweight on his desk with density xz. The paperweight is a tetrahedron with corners $(0, 0, 0), (0, 1, 0), (1, 1, 0), (0, 1, 1)$.**
   (a) (7 points) **Set up a triple integral with differential $\mathrm{d}V = \mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$ that represents the mass of the paperweight P.**
   (b) (7 points) **Solve the integral from part (a) to compute the mass of the paperweight.**

5. (11 points) **Let X and Y be two jointly continuous random variables with joint PDF**

$$f_{XY}(x, y) = \begin{cases} x + cy^2, & 0 \leq x, y \leq 1 \\ 0, & \textbf{elsewhere} \end{cases}$$

   (a) (4 points) Set up the integral representing the probability
   (b) (7 points) Solve for c.

6. **Let $f(x) = e^{-3x}$.**
   (a) (10 points) **Write a Taylor Series for f centered at $x = -2$.**
   (b) (5 points) **Compute the interval of convergence of your series in part (a). Name any tests or theorems that you used. You do *NOT* need to specify whether the seriese converges at the endpoints.**
   (c) (5 points) **Suppose you wanted to use the Taylor polynomial $T_5(x)$ (still centered at -2) to approximate e^4. Find a bound on the error of this approximation. You do not need to simplify the arithmetic in your answer.**

End of exam.