

Homework # 0: Matrix Operations

Solutions

1. **RREF** Find the reduced row echelon form of the following matrix:

$$\mathbf{A} = \begin{pmatrix} 1 & 5 & 6 \\ 2 & 7 & 3 \end{pmatrix}$$

Check your answer using the MATLAB command `rref(A)`, where $\mathbf{A}$ is the matrix.

Solution. First, let's contextualize this problem. $\mathbf{A}$ is an augmented matrix arising from the system of linear equations

$$\begin{aligned} x + 5y &= 6 \\ 2x + 7y &= 3 \end{aligned}$$

The coefficient matrix is $\begin{pmatrix} 1 & 5 \\ 2 & 7 \end{pmatrix}$ and the right hand side is $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$. By concatenating the coefficient and right hand side matrices, we end up with $\mathbf{A}$. Now we need to perform elementary row operations to get this matrix into RREF.

$$\begin{aligned} \begin{pmatrix} 1 & 5 & 6 \\ 2 & 7 & 3 \end{pmatrix} &\xrightarrow{2R_1 - R_2 \rightarrow R_2} \begin{pmatrix} 1 & 5 & 6 \\ 0 & 3 & 9 \end{pmatrix} \\ &\xrightarrow{1/3R_2 \rightarrow R_2} \begin{pmatrix} 1 & 5 & 6 \\ 0 & 1 & 3 \end{pmatrix} \\ &\xrightarrow{-5R_2 + R_1 \rightarrow R_1} \begin{pmatrix} 1 & 0 & -9 \\ 0 & 1 & 3 \end{pmatrix} \end{aligned}$$

to double check our computations, we can use MATLAB.

we see in Code block 1 how to write a program that would solve the answer.

As we see from the output, that our answer is correct.

```

1 % Code to find the reduced Row Eschelon Form
2 function sol0q1()
3     A = [ 1,5,6; 2,7,3];
4     sol = rref(A)
5 end
6
7 >> sol0q0
8
9 sol =
10
11 1      0      -9
12 0      1       3

```

Figure 1: Code to compute the RREF of a matrix

2. Adding and Subtracting Matrices Let

$$\mathbf{A} = \begin{pmatrix} 6 & 0 \\ 1 & 5 \\ 3 & 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 & 3 \\ 5 & 6 \\ 3 & 1 \end{pmatrix}.$$

Find $3\mathbf{A} - 2\mathbf{B}$. Show all work. Additionally, check your answer with MATLAB.

Solution. This problem encapsulates scalar multiplication and matrix addition/subtraction. First let's find what $3\mathbf{A}$ and $2\mathbf{B}$ are.

$$3\mathbf{A} = 3 \begin{pmatrix} 6 & 0 \\ 1 & 5 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 \cdot 6 & 3 \cdot 0 \\ 3 \cdot 1 & 3 \cdot 5 \\ 3 \cdot 3 & 3 \cdot 1 \end{pmatrix} = \begin{pmatrix} 18 & 0 \\ 3 & 15 \\ 9 & 3 \end{pmatrix}$$

$$2\mathbf{B} = 2 \begin{pmatrix} 1 & 3 \\ 5 & 6 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 2 \cdot 1 & 2 \cdot 3 \\ 2 \cdot 5 & 2 \cdot 6 \\ 2 \cdot 3 & 2 \cdot 1 \end{pmatrix} = \begin{pmatrix} 2 & 6 \\ 10 & 12 \\ 6 & 2 \end{pmatrix}$$

Now we need to compute $3\mathbf{A} - 2\mathbf{B}$, which can be done elementwise,

meaning $c_{ij} = 3a_{ij} - 2b_{ij}$ for all rows i and all columns j .

$$\begin{aligned}\mathbf{C} &= 3\mathbf{A} - 2\mathbf{B} \\ &= \begin{pmatrix} 18 & 0 \\ 3 & 15 \\ 9 & 3 \end{pmatrix} - \begin{pmatrix} 2 & 6 \\ 10 & 12 \\ 6 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 18-2 & 0-6 \\ 3-10 & 15-12 \\ 9-6 & 3-2 \end{pmatrix} \\ &= \begin{pmatrix} 16 & -6 \\ -7 & 3 \\ 3 & 1 \end{pmatrix}\end{aligned}$$

Again we can check this by using MATLAB.

```

1 % Code to find 3A - 2B
2 function sol0q2()
3     A = [6,0;1,5;3,1];
4     B = [1,3;5,6;3,1];
5
6     sol = 3*A - 2*B
7 end
8
9 >> sol0q1
10
11 sol =
12
13     16     -6
14     -7      3
15      3      1

```

Figure 2: Code to compute matrix addition and scalar multiplication

3. Matrix Multiplication Let

$$\mathbf{A} = \begin{pmatrix} 5 & 2 \\ 5 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 3 & -1 & 0 \\ 2 & 1 & 5 \end{pmatrix}.$$

Now let $\mathbf{C} = \mathbf{AB}$. Before doing the computation, justify what the matrix dimensions of $\mathbf{C}$ are, if multiplication is well defined. Then compute $\mathbf{C}$. Check your work with MATLAB.

Solution. First, we note that $\mathbf{A}$ has two rows and 2 columns (which we might abbreviate as $\mathbb{R}^{2 \times 2}$, where the $\mathbb{R}$ means real numbers.). Similarly, $\mathbf{B}$ has 2 rows and 3 columns (so $\mathbb{R}^{2 \times 3}$). Since we are doing $\mathbf{C} = \mathbf{AB}$, then we know the only way that matrix multiplication can happen is if the number of columns of $\mathbf{A}$ match the number of rows of $\mathbf{B}$. Since $\mathbf{A}$ is in $\mathbb{R}^{2 \times 2}$, it has 2 columns, and $\mathbf{B}$ is in $\mathbb{R}^{2 \times 3}$, it has two rows, so this means the matrix multiplication will work. The output of the matrix multiplication will be 2 by 3. Each of the column and row operations, we need to do one for each row of $\mathbf{A}$ and for each column of $\mathbf{B}$.

$$\begin{aligned} \mathbf{C} &= \mathbf{AB} \\ &= \begin{pmatrix} 5 & 2 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} 3 & -1 & 0 \\ 2 & 1 & 5 \end{pmatrix} \\ &= \begin{pmatrix} 5 \cdot 3 + 2 \cdot 2 & 5 \cdot (-1) + 2 \cdot 1 & 5 \cdot 0 + 2 \cdot 5 \\ 5 \cdot 3 + 3 \cdot 2 & 5 \cdot (-1) + 3 \cdot 1 & 5 \cdot 0 + 3 \cdot 5 \end{pmatrix} \\ &= \begin{pmatrix} 19 & -3 & 10 \\ 21 & -2 & 15 \end{pmatrix} \end{aligned}$$

We check this with MATLAB

```

1  % Code to find 3A - 2B
2  function sol0q3()
3      A = [5,2;5,3];
4      B = [3,-1,0;2,1,5];
5
6      sol = A*B
7  end
8
9  >> sol0q3
10
11 sol =
12
13      19      -3      10
14      21      -2      15

```

Figure 3: Code to compute matrix addition and scalar multiplication

4. **Matrix multiplication is not commutative!** In most math you have seen before, if I gave you two numbers, $a = 4, b = 3$, then $ab = (4)(3) = 12 = (3)(4) = ba$. We call this “commutativity”. However, in class,

I mentioned that matrices and matrix multiplication doesn't have this property. Let's see this through.

Let

$$\mathbf{A} = \begin{pmatrix} 4 & -1 & 0 \\ 4 & 0 & 5 \\ 2 & 6 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 6 & -1 & 5 \\ -1 & 6 & 5 \\ 2 & -1 & 5 \end{pmatrix}.$$

Compute $\mathbf{AB}$, compute $\mathbf{BA}$, and comment on if these two are equal. Check your work with MATLAB.

Solution. First let's compute $\mathbf{AB}$, where we will use different colors to differentiate between the two.

$$\begin{aligned} \mathbf{AB} &= \begin{pmatrix} 4 & -1 & 0 \\ 4 & 0 & 5 \\ 2 & 6 & 3 \end{pmatrix} \begin{pmatrix} 6 & -1 & 5 \\ -1 & 6 & 5 \\ 2 & -1 & 6 \end{pmatrix} \\ &= \begin{pmatrix} 4 \cdot 6 + (-1) \cdot (-1) + 0 \cdot 2 & 4 \cdot (-1) + (-1) \cdot 6 + 0 \cdot (-1) & 4 \cdot 5 + (-1) \cdot 5 + 0 \cdot 6 \\ 4 \cdot 6 + 0 \cdot (-1) + 5 \cdot 2 & 4 \cdot (-1) + 0 \cdot 6 + 5 \cdot (-1) & 4 \cdot 5 + 0 \cdot 5 + 5 \cdot 6 \\ 2 \cdot 6 + 6 \cdot (-1) + 3 \cdot 2 & 2 \cdot (-1) + 6 \cdot 6 + 3 \cdot (-1) & 2 \cdot 5 + 6 \cdot 5 + 3 \cdot 6 \end{pmatrix} \\ &= \begin{pmatrix} 24 + 1 + 0 & -4 - 6 + 0 & 20 - 5 + 0 \\ 24 + 0 + 10 & -4 + 0 - 5 & 20 + 0 + 30 \\ 12 - 6 + 6 & -2 + 36 - 3 & 10 + 30 + 18 \end{pmatrix} \\ &= \begin{pmatrix} 25 & -10 & 15 \\ 34 & -9 & 50 \\ 12 & 31 & 58 \end{pmatrix} \end{aligned}$$

Now we compute $\mathbf{BA}$.

$$\begin{aligned} \mathbf{BA} &= \begin{pmatrix} 6 & -1 & 5 \\ -1 & 6 & 5 \\ 2 & -1 & 6 \end{pmatrix} \begin{pmatrix} 4 & -1 & 0 \\ 4 & 0 & 5 \\ 2 & 6 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 6 \cdot 4 + (-1) \cdot 4 + 5 \cdot 2 & 6 \cdot (-1) + (-1) \cdot 0 + 5 \cdot 6 & 6 \cdot 0 + (-1) \cdot 5 + 5 \cdot 3 \\ -1 \cdot 4 + 6 \cdot 4 + 5 \cdot 2 & -1 \cdot (-1) + 6 \cdot 0 + 5 \cdot 6 & -1 \cdot 0 + 6 \cdot 5 + 5 \cdot 3 \\ 2 \cdot 4 + (-1) \cdot 4 + 6 \cdot 2 & 2 \cdot (-1) + (-1) \cdot 0 + 6 \cdot 6 & 2 \cdot 0 + (-1) \cdot 5 + 6 \cdot 3 \end{pmatrix} \\ &= \begin{pmatrix} 24 - 4 + 10 & -6 + 0 + 30 & 0 - 5 + 15 \\ -4 + 24 + 10 & 1 + 0 + 30 & 0 + 30 + 15 \\ 8 - 4 + 12 & -2 + 0 + 36 & 0 - 5 + 18 \end{pmatrix} \\ &= \begin{pmatrix} 30 & 24 & 10 \\ 30 & 31 & 45 \\ 16 & 34 & 13 \end{pmatrix} \end{aligned}$$

As we can see, $\mathbf{AB} \neq \mathbf{BA}$, so matrices are not commutative. Even more, every element is different between the two!

Using MATLAB, we see that we are correct, and using `same` we see that every single element in $\mathbf{AB}$ is different from $\mathbf{BA}$, since `0=False`.

5. **Determinants of small matrices** Find the determinants of the following matrices:

(a)

$$\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 7 & 4 \end{pmatrix}$$

Solution. Remember we have a really nice formula for the determinant of a 2×2 matrix.

$$\det(\mathbf{A}) = \begin{vmatrix} 3 & 2 \\ 7 & 4 \end{vmatrix} = 3(4) - 2(7) = 12 - 14 = -2$$

We get to verify this using MATLAB, the code is given below in code block 5.

(b)

$$\mathbf{B} = \begin{pmatrix} -1 & 1 & 2 \\ 5 & 6 & 2 \\ 4 & -1 & 5 \end{pmatrix}$$

Solution. The determinant of a 3×3 matrix has a formula, albeit not as nice as the 2×2 case, but in fact, uses the 2×2 determinant in it.

$$\begin{aligned} \det(\mathbf{B}) &= -1 \begin{vmatrix} 6 & 2 \\ -1 & 5 \end{vmatrix} - 1 \begin{vmatrix} 5 & 2 \\ 4 & 5 \end{vmatrix} + 2 \begin{vmatrix} 5 & 6 \\ 4 & -1 \end{vmatrix} \\ &= -(6 \cdot 5 - 2 \cdot (-1)) - (5 \cdot 5 - 2 \cdot 4) + 2(5 \cdot (-1) - 6 \cdot 4) \\ &= -30 - 2 - 25 + 8 - 10 - 48 \\ &= -107 \end{aligned}$$

We can check our answer with MATLAB, which is given in code block 6.

6. **Area of a Triangle via Determinants** Below, you will see a triangle with vertices on $(0, 0)$, $(1, 2)$, and $(4, 0)$. We can think of this as the area spanned by two vectors, $\langle 4, 0 \rangle$ and $\langle 1, 2 \rangle$. One of the applications of the determinant tells us the signed area of the parallelogram of the two vectors. Using these facts, compute the area (should be positive) of the triangle using a determinant. Compare your solution to using the formula $A_{\Delta} = \frac{1}{2}bh$, to find the area.

```
1 function sol0q4()
2     % Code to see if matrices commute (they do not)
3     A = [4,-1,0;4,0,5;2,6,3];
4     B = [6,-1,5;-1,6,5;2,-1,6];
5
6     solAB = A * B
7     solBA = B * A
8
9     same = (solAB == solBA)
10 end
11
12 >> sol0q4
13
14 solAB =
15
16         25    -10     15
17         34     -9     50
18         12     31     58
19
20
21 solBA =
22
23         30     24     10
24         30     31     45
25         16     34     13
26
27
28 same =
29
30 3x3 logical array
31
32         0     0     0
33         0     0     0
34         0     0     0
```

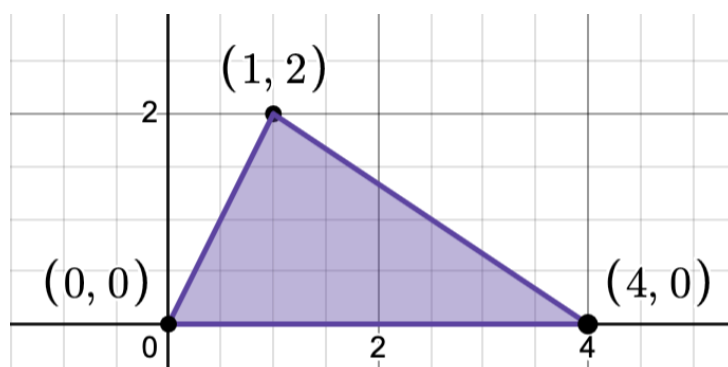
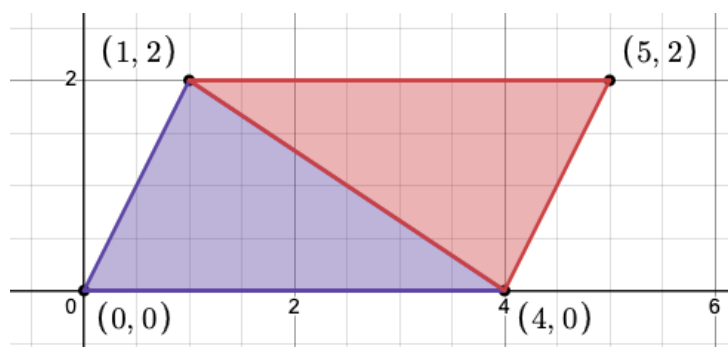
Figure 4: Code to compute matrix addition and scalar multiplication

```
1 function sol0q5a()
2     % Code to see matrix determinant
3     A = [3,2;7,4];
4     det(A)
5
6 end
7
8 >> sol0q5a
9
10 sol =
11     -2.0000
```

Figure 5: Code to compute 2×2 determinant

```
1 function sol0q5b()
2     % Code to see matrix determinant
3     B = [-1,1,2;5,6,2;4,-1,5];
4     det(B)
5
6 end
7
8 >> sol0q5b
9
10 sol =
11     -107
```

Figure 6: Code to compute 3×3 determinant

Figure 7: A triangle with one corner on $(0,0)$.Figure 8: A parallelogram with one corner on $(0,0)$ that is two triangles stacked together.

Solution. Since we know that the determinant of two vectors gives the area of a parallelogram, we must divide the determinant by 2 to get the area of the purple triangle. We construct the matrix $\mathbf{A} = \begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix}$, being the two vectors concatenated on top of each other. From this we have

$$\begin{aligned} \det(\mathbf{A}) &= \det \begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix} \\ &= 4 \cdot 2 - 1 \cdot 0 = 8 - 0 = 8. \end{aligned}$$

As mentioned above, this means the parallelogram (depicted in figure 8) has area 8 units squared.

This means $A_{\triangle} = \frac{1}{2} \det(\mathbf{A}) = \frac{1}{2} 8 = 4$ units squared from the determinant approach.

This aligns with the typical area formula for a triangle. The base has length 4 ($4 - 0 = 4$ on the x -axis) and the height, which is perpendicular

to the x -axis has height 2 units, so $A_{\triangle} = \frac{1}{2}bh = \frac{1}{2}(4)(2) = 4$ units squared.

Remark. I really like this question because you cannot go wrong! If you decided to construct the matrix $\tilde{\mathbf{A}} = \begin{pmatrix} 1 & 2 \\ 4 & 0 \end{pmatrix}$, i.e., you switched the rows, the determinant would have been $\det(\tilde{\mathbf{A}}) = 1 \cdot 0 - 2 \cdot 4 = -8$, but I said that the area had to be positive (also most of the time, a negative area doesn't make sense). This also makes sense given the notes where we know the determinant switches signs when you swap rows.

If you would have constructed $\hat{\mathbf{A}} = \begin{pmatrix} 4 & 1 \\ 0 & 2 \end{pmatrix}$, i.e., put the vectors as columns instead of rows, we still would have found $\det(\hat{\mathbf{A}}) = 8 - 0 = 8$. (This comes from a fact that wasn't touched on in class that $\det(\mathbf{A}) = \det(\mathbf{A}^{\top})$, where $\cdot^{\top}$ is the matrix transpose.)

Lastly, if you would have both placed the vectors as columns and considered $\langle 1, 2 \rangle$ to be the first column, you would have still gotten $|-8|$, which would have lead you to the same answer.

7. **Inverse of a 2×2 matrix** Use the formula presented in class to find $\mathbf{A}^{-1}$ where

$$\mathbf{A} = \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}.$$

Show your work that this is truly the inverse of $\mathbf{A}$ by computing BOTH $\mathbf{A}\mathbf{A}^{-1}$ and $\mathbf{A}^{-1}\mathbf{A}$. What do they both equal?

Solution. Using the formula we presented in class,

$$\begin{aligned} \mathbf{A}^{-1} &= \frac{1}{\det(\mathbf{A})} \begin{pmatrix} 2 & 1 \\ 0 & 4 \end{pmatrix} \\ &= \frac{1}{4(2) + 1(0)} \begin{pmatrix} 2 & 1 \\ 0 & 4 \end{pmatrix} \\ &= \frac{1}{8} \begin{pmatrix} 2 & 1 \\ 0 & 4 \end{pmatrix} \end{aligned}$$

Now we compute $\mathbf{A}\mathbf{A}^{-1}$

$$\begin{aligned}\mathbf{A}\mathbf{A}^{-1} &= \frac{1}{8} \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 0 & 4 \end{pmatrix} \\ &= \frac{1}{8} \begin{pmatrix} 4 \cdot 2 + (-1) \cdot 0 & 4 \cdot 1 + (-1) \cdot 4 \\ 0 \cdot 2 + 2 \cdot 0 & 0 \cdot 1 + 2 \cdot 4 \end{pmatrix} \\ &= \frac{1}{8} \begin{pmatrix} 8 + 0 & 4 - 4 \\ 0 + 0 & 0 + 8 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\end{aligned}$$

Now we compute $\mathbf{A}^{-1}\mathbf{A}$.

$$\begin{aligned}\mathbf{A}^{-1}\mathbf{A} &= \frac{1}{8} \begin{pmatrix} 2 & 1 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix} \\ &= \frac{1}{8} \begin{pmatrix} 2 \cdot 4 + 1 \cdot 0 & 2 \cdot (-1) + 1 \cdot 2 \\ 0 \cdot 4 + 4 \cdot 0 & 0 \cdot (-1) + 4 \cdot 2 \end{pmatrix} \\ &= \frac{1}{8} \begin{pmatrix} 8 + 0 & -2 + 2 \\ 0 + 0 & 0 + 8 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\end{aligned}$$

Both $\mathbf{A}\mathbf{A}^{-1} = \mathbf{I} = \mathbf{A}^{-1}\mathbf{A}$.

8. Inverse of a 3×3 matrix Let

$$\mathbf{A} = \begin{pmatrix} 0 & -3 & -2 \\ 1 & -4 & -2 \\ -3 & 4 & 1 \end{pmatrix}.$$

One way to find the inverse of this matrix, $\mathbf{A}^{-1}$ is to augment $\mathbf{A}$ with the identity matrix, $\mathbf{I}_3$, meaning $(\mathbf{A} \mid \mathbf{I}_3)$. This would result in

$$\left(\begin{array}{ccc|ccc} 0 & -3 & -2 & 1 & 0 & 0 \\ 1 & -4 & -2 & 0 & 1 & 0 \\ -3 & 4 & 1 & 0 & 0 & 1 \end{array} \right).$$

From this, we would use elementary matrix row operations to perform Gaussian Elimination. This would take $\mathbf{A}$ to $\mathbf{I}$. And when you do this operation for the augmented matrix, you end up with $(\mathbf{I}_3 \mid \mathbf{A}^{-1})$. Using this technique, find $\mathbf{A}^{-1}$.

Check your answer using the MATLAB command `inv(A)`, where $\mathbf{A}$ is the matrix.

Solution.

$$\begin{aligned}
 & \left(\begin{array}{ccc|ccc} 0 & -3 & -2 & 1 & 0 & 0 \\ 1 & -4 & -2 & 0 & 1 & 0 \\ -3 & 4 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & 0 & 0 \\ -3 & 4 & 1 & 0 & 0 & 1 \end{array} \right) \\
 & \xrightarrow{3R_1 + R_3 \rightarrow R_3} \left(\begin{array}{ccc|ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & 0 & 0 \\ 0 & -8 & -5 & 0 & 3 & 1 \end{array} \right) \\
 & \xrightarrow{-1/3 R_2 \rightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{-1}{3} & 0 & 0 \\ 0 & -8 & -5 & 0 & 3 & 1 \end{array} \right) \\
 & \xrightarrow{8R_2 + R_3 \rightarrow R_3} \left(\begin{array}{ccc|ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{-1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{-8}{3} & 3 & 1 \end{array} \right) \\
 & \xrightarrow{4R_2 + R_1 \rightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{3} & \frac{-4}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{-1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{-8}{3} & 3 & 1 \end{array} \right) \\
 & \xrightarrow{-2R_3 + R_2 \rightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{3} & \frac{-4}{3} & 1 & 0 \\ 0 & 1 & 0 & 5 & -6 & -2 \\ 0 & 0 & \frac{1}{3} & \frac{-8}{3} & 3 & 1 \end{array} \right) \\
 & \xrightarrow{-2R_3 + R_1 \rightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & -5 & -2 \\ 0 & 1 & 0 & 5 & -6 & -2 \\ 0 & 0 & \frac{1}{3} & \frac{-8}{3} & 3 & 1 \end{array} \right) \\
 & \xrightarrow{3R_3 \rightarrow R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & -5 & -2 \\ 0 & 1 & 0 & 5 & -6 & -2 \\ 0 & 0 & 1 & -8 & 9 & 3 \end{array} \right)
 \end{aligned}$$

This was a lot of computation, so to verify our answers, we use MATLAB to check, using code in code block 9.

Another way to compute this would be to combine skills we used in question 1 and the problem statement here. We need to construct $\mathbf{A}$ and then tack on $\mathbf{I}_3$ after it. Then we performed `rref` on this matrix. This is preferable to using the `inv` function for very complicated reasons, but as a numerical linear algebraist, I cannot advocate for the `inv` function. As you see from the code in code block 10, the last 3 columns gives the same matrix as above.

9. **Reflection** How long did this assignment take you (be honest)? Do you feel that this is too short or long?

```
1 function sol0q8a()
2     % Code to compute the inverse of a matrix
3     A = [0, -3, -2; 1, -4, -2; -3, 4, 1];
4     sol = inv(A)
5 end
6
7 >> sol0q8a
8
9 sol =
10
11     4.0000    -5.0000    -2.0000
12     5.0000    -6.0000    -2.0000
13    -8.0000     9.0000     3.0000
```

Figure 9: Code to compute the inverse of a matrix

```
1 function sol0q8b()
2     % Code to compute the inverse of a
3     % matrix
4     A = [0, -3, -2; 1, -4, -2; -3, 4, 1];
5     I = eye(3);
6     AI = [A, I];
7     rref(AI)
8 end
9
10 >> sol0q8b
11
12 sol =
13     1     0     0     4    -5    -2
14     0     1     0     5    -6    -2
15     0     0     1    -8     9     3
```

Figure 10: Code to compute the the RREF of a matrix of interest concatenated with the Identity

Solution. Answers will vary. I am just trying to gauge the level of work I am assigning. Emory policy says that this class should have 150 minutes of in class instruction and 300 minutes of outside work per week. My expectation is to make sure we aren't going anywhere near that.

This is the first time this course is being taught, so I am interested in how it is going on your end. I am always open to feedback to make your experience better. Either tell me, or if you don't feel comfortable, please reach out to Prof. Ettinger.