Homework # 6

Markov Chains: Long Run Behavior and Exits

SP §1.4

1. **Find the stationary distributions for the Markov chains with transition matrices:**

(a)	**1**	**2**	**3**
1	0.5	0.4	0.1
2	0.2	0.5	0.3
3	0.1	0.3	0.6

(b)	**1**	**2**	**3**
1	0.5	0.4	0.1
2	0.3	0.4	0.3
3	0.2	0.2	0.6

(c)	**1**	**2**	**3**
1	0.6	0.4	0.0
2	0.2	0.4	0.2
3	0.0	0.2	0.8

2. **Find the stationary distributions for the Markov chains on $\{1, 2, 3, 4\}$ with transition matrices:**

(a) $\begin{pmatrix} 0.7 & 0.0 & 0.3 & 0.0 \\ 0.6 & 0.0 & 0.4 & 0.0 \\ 0.0 & 0.5 & 0.0 & 0.5 \\ 0.0 & 0.4 & 0.0 & 0.6 \end{pmatrix}$ (b) $\begin{pmatrix} 0.7 & 0.3 & 0.0 & 0.0 \\ 0.2 & 0.5 & 0.3 & 0.0 \\ 0.0 & 0.3 & 0.6 & 0.1 \\ 0.0 & 0.0 & 0.2 & 0.8 \end{pmatrix}$ (c) $\begin{pmatrix} 0.7 & 0.0 & 0.3 & 0.0 \\ 0.2 & 0.5 & 0.3 & 0.0 \\ 0.1 & 0.2 & 0.4 & 0.3 \\ 0.0 & 0.4 & 0.0 & 0.6 \end{pmatrix}$

SP §1.5

3. **Consider the Markov chain with transition matrix:**

	1	**2**	**3**	**4**
1	0.0	0.0	0.1	0.9
2	0.0	0.0	0.6	0.4
3	0.8	0.2	0.0	0.0
4	0.4	0.6	0.0	0.0

   (a) **Compute p^2.**

   (b) **Find the stationary distributions of p and all of the stationary distributions of p^2.**

   (c) **Find the limit of $p^{2n}(x, x)$ as $n \to \infty$.**

4. **Do the following Markov chains converge to equilbrium?**

(a)	**1**	**2**	**3**	**4**
1	0.0	0.0	1.0	0.0
2	0.0	0.0	0.5	0.5
3	0.3	0.7	0.0	0.0
4	1.0	0.0	0.0	0.0

(b)	**1**	**2**	**3**	**4**
1	0.0	1.0	0.0	0.0
2	0.0	0.0	0.0	1.0
3	1.0	0.0	0.0	0.0
4	1/3	0.0	2/3	0.0

(c)	**1**	**2**	**3**	**4**	**5**	**6**
1	0.0	0.5	0.5	0.0	0.0	0.0
2	0.0	0.0	0.0	1.0	0.0	0.0
3	0.0	0.0	0.0	0.4	0.0	0.6
4	1.0	0.0	0.0	0.0	0.0	0.0
5	0.0	1.0	0.0	0.0	0.0	0.0
6	0.2	0.0	0.0	0.0	0.8	0.0

SP §1.8

5. **A bank classifies loans as paid in full (F), in good standing (G), in arrears (A), or as a bad debt (B). Loans move between the categories according to the following transition probability:**

	F	**G**	**A**	**B**
F	1.0	0.0	0.0	0.0
G	0.1	0.8	0.1	0.0
A	0.1	0.4	0.4	0.1
B	1.0	0.0	0.0	1.0

What fraction of loans in good standing are eventually paid in full? What is the answr for those in arrears?

6. **Use the second solution in Example 1.48 to compute the expected waiting times for the patterns HHH, HHT, HTT, and HTH. Which pattern has the longest waiting time? Which ones achieve the minimum value of 8?**