

Homework # 5: Markov Chains: Definitions and Examples

Due: April 15, 11:30 am

1. **Polya's Urn** Five white balls and five black balls are distributed in two urns in such a way that each urn contains five balls. At each step we draw one ball from each urn and exchange them. Let X_n be the number of white balls in the left urn at time n . Compute the transition probability for X_n .
2. **Homeownership** The 1990 census showed that 36% of the households in the District of Columbia were homeowners while the remainder were renters. During the next decade 6% of the homeowners became renters and 12% of the renters became homeowners. What percentage were homeowners in 2000? in 2010?
3. **Taxi driver** A taxicab driver moves between the airport A and two hotels B and C according to the following rules. If he is at the airport, he will be at one of the two hotels next with equal probability. If at a hotel then he returns to the airport with probability $3/4$ and goes to the other hotel with probability $1/4$.
 - (a) Find the transition matrix for the chain.
 - (b) Suppose the driver begins at the airport at time 0. Find the probability for each of his three possible locations at time 2 and the probability he is at hotel B at time 3.
4. **Matrix States** Consider the following transition matrices. Identify the transient and recurrent states, and the irreducible closed sets in the Markov chains. Give reasons for your answers.

(a)	1	2	3	4	5	(b)	1	2	3	4	5	6
1	0.4	0.3	0.3	0	0	1	0.1	0	0	0.4	0.5	0
2	0	0.5	0	0.5	0	2	0.1	0.2	0.2	0	0.5	0
3	0.5	0	0.5	0	0	3	0	0.1	0.3	0	0	0.6
4	0	0.5	0	0.5	0	4	0.1	0	0	0.9	0	0
5	0	0.3	0	0.3	0.4	5	0	0	0	0.4	0	0.6
						5	0	0	0	0	0.5	0.5

5. **Reflection** How long did this assignment take you (be honest)? Do you feel that this is too short or long? How is the material going? How can I help you?