MATH 190: MATHEMATICS IN SUSTAINABILITY
Emory University
Spring 2026
Prof. Mitchell Scott

Homework # 1: Decision Problems and Static Games

Due: February 11, 11:30 am

1. **Fruit Trees** You have room for up to two fruit-bearing trees in your garden. The fruit trees that can grow in your garden are either apple, orange, or pear. The cost of maintenance is \$100 for an apple tree, \$70 for an orange tree, and \$120 for a pear tree. Your food bill will be reduced by \$130 for each apple tree you plant, by \$145 for each pear tree you plant, and by \$90 for each orange tree you plant. You care only about your total expenditure in making any planting decisions.

   (a) What is the set of possible actions and related outcomes?

   (b) What is the payoff of each action/outcome?

   (c) Draw the associated decision tree. What will a rational player choose?

   (d) Now imagine that the reduction in your food bill is half for the second tree of the same kind. (You like variety.) That is, the first apple tree still reduces your food bill by \$130, but if you plant two apple trees your food bill will be reduced by \$130 + \$65 = \$195, and similarly for pear and orange trees. What will a rational player choose now?

2. **Discount Prices** A local department store sells products at a given initial price, and every week a product goes unsold, its price is discounted by 25% of the original price. If it is not sold after four weeks, it is sent back to the regional warehouse. A set of kitchen knives was just put out for \$200. Your willingness to pay for the knives (your dollar value) is \$180, so if you buy them at a price P, your payoff is $u = 180 - P$. If you don't buy the knives, the chances that they will be sold to someone else, conditional on not having been sold the week before, are as follows

Week 1	0.2
Week 2	0.4
Week 3	0.6
Week 4	0.8

For example, if you do not buy the knives during the first two weeks, the likelihood that they will be available at the beginning of the third week is the likelihood that they do not sell in either week 1 or week 2, which is $0.8 \times 0.6 = 0.48$.

(a) Draw your decision tree for the four weeks after the knives are put out for sale.

(b) At the beginning of which week, if any, should you run to buy the knives?

(c) Find a willingness to pay for the knives that would make it optimal to buy at the beginning of the first week.

(d) Find a willingness to pay that would make it optimal to buy at the beginning of the fourth week.

3. **Matching Pennies** Players 1 and 2 each put a penny on a table simultaneously. If the two pennies come up the same side (heads or tails) then player 1 gets both pennies; otherwise player 2 gets both pennies. Represent this game as a matrix.

4. **Public Good Contribution** Three players live in a town, and each can choose to contribute to fund a streetlamp. The value of having the streetlamp is 3 for each player, and the value of not having it is 0. The mayor asks each player to contribute either 1 or nothing. If at least two players contribute then the lamp will be erected. If one player or no players contribute then the lamp will not be erected, in which case any person who contributed will not get his money back. Write down the normal form of this game.

5. **Reflection** How long did this assignment take you (be honest)? Do you feel that this is too short or long? How is the material going? How can I help you?