Math 190: Mathematics in Sustainability
Emory University
Spring 2026
Prof. Mitchell Scott

Homework # 0: Matrix Operations

Due: January 28, 11:30 am

1. **RREF** Find the reduced row echelon form of the following matrix:

$$\mathbf{A} = \begin{pmatrix} 1 & 5 & 6 \\ 2 & 7 & 3 \end{pmatrix}$$

Check your answer using the Matlab command `rref(A)`, where `A` is the matrix.

2. **Adding and Subtracting Matrices** Let

$$\mathbf{A} = \begin{pmatrix} 6 & 0 \\ 1 & 5 \\ 3 & 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 & 3 \\ 5 & 6 \\ 3 & 1 \end{pmatrix}.$$

Find $3\mathbf{A} - 2\mathbf{B}$. Show all work. Additionally, check your answer with Matlab.

3. **Matrix Multiplication** Let

$$\mathbf{A} = \begin{pmatrix} 5 & 2 \\ 5 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 3 & -1 & 0 \\ 2 & 1 & 5 \end{pmatrix}.$$

Now let $\mathbf{C} = \mathbf{AB}$. Before doing the computation, justify what the matrix dimensions of $\mathbf{C}$ are, if multiplication is well defined. Then compute $\mathbf{C}$. Check your work with Matlab.

4. **Matrix multiplication is not commative!** In most math you have seen before, if I gave you two numbers, $a = 4, b = 3$, then $ab = (4)(3) = 12 = (3)(4) = ba$. We call this "commutativity". However, in class, I mentioned that matrices and matrix multiplication doesn't have this property. Let's see this through.

Let

$$\mathbf{A} = \begin{pmatrix} 4 & -1 & 0 \\ 4 & 0 & 5 \\ 2 & 6 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 6 & -1 & 5 \\ -1 & 6 & 5 \\ 2 & -1 & 5 \end{pmatrix}.$$

Compute **AB**, compute **BA**, and comment on if these two are equal. Check your work with MATLAB.

5. **Determinants of small matrices** Find the determinants of the following matrices:

   (a)
   $$\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 7 & 4 \end{pmatrix}$$

   (b)
   $$\mathbf{B} = \begin{pmatrix} -1 & 1 & 2 \\ 5 & 6 & 2 \\ 4 & -1 & 5 \end{pmatrix}$$

6. **Area of a Triangle via Determinants** Below, you will see a triangle with vertices on $(0,0), (1,2)$, and $(4,0)$. We can think of this as the

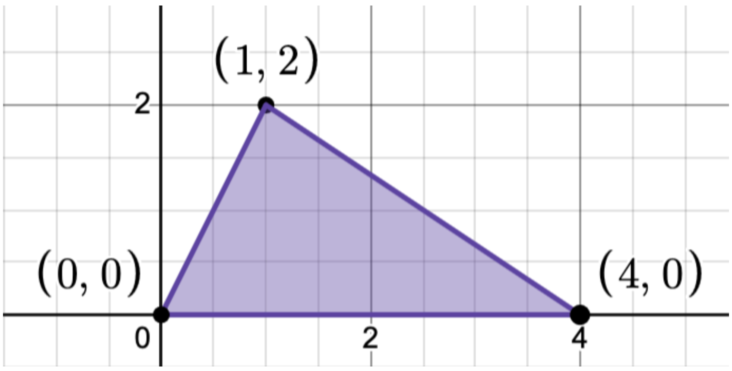

Figure 1: A triangle with one corner on $(0,0)$.

area spanned by two vectors, $\langle 4,0\rangle$ and $\langle 1,2\rangle$. One of the applications of the determinant tells us the signed area of the parallelogram of the two vectors. Using these facts, compute the area (should be positive) of the triangle using a determinant. Compare your solution to using the formula $A_\triangle = \frac{1}{2}bh$, to find the area.

7. **Inverse of a 2×2 matrix** Use the formula presented in class to find $\mathbf{A}^{-1}$ where
$$\mathbf{A} = \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}.$$

Show your work that this is truly the inverse of $\mathbf{A}$ by computing BOTH $\mathbf{A}\mathbf{A}^{-1}$ and $\mathbf{A}^{-1}\mathbf{A}$. What do they both equal?

8. **Inverse of a 3×3 matrix** Let

$$\mathbf{A} = \begin{pmatrix} 0 & -3 & -2 \\ 1 & -4 & -2 \\ -3 & 4 & 1 \end{pmatrix}.$$

One way to find the inverse of this matrix, $\mathbf{A}^{-1}$ is to augment $\mathbf{A}$ with the identity matrix, $\mathbf{I}_3$, meaning $\left(\mathbf{A} \mid \mathbf{I}_3\right)$. This would result in

$$\left(\begin{array}{ccc|ccc} 0 & -3 & -2 & 1 & 0 & 0 \\ 1 & -4 & -2 & 0 & 1 & 0 \\ -3 & 4 & 1 & 0 & 0 & 1 \end{array}\right).$$

From this, we would use elementary matrix row operations to perform Gaussian Elimination. This would take $\mathbf{A}$ to $\mathbf{I}$. And when you do this operation for the augmented matrix, you end up with $\left(\mathbf{I}_3 \mid \mathbf{A}^{-1}\right)$. Using this technique, find $\mathbf{A}^{-1}$.

Check your answer using the MATLAB command `inv(A)`, where `A` is the matrix.

9. **Reflection** How long did this assignment take you (be honest)? Do you feel that this is too short or long?