Emory University
Final Exam Review: L'Hopitals, Linear Approximation, and Curve Sketching
MA-111 Calculus I
Date: December 15, 2025
Instructor: Mitchell Scott

Student ID _____ Name: _____

Please read the following instructions carefully.

- This is a review sheet, so there is no time limit. This question booklet contains 8 questions, 9 pages (including the cover) for the total of 80 points/marks. Check to see if any pages are missing. DO NOT scribble or do rough work or make any stray marks on it. Use separate sheet for rough work.
- All the questions are compulsory and all the notations have their usual meaning. You can use standard results directly by writing its statement appropriately. **Read the instructions for individual questions carefully** before answering the questions.
- These questions were developed by me and not overseen by other instructors or the head coordinator.

Question	Points	Score
1	4	
2	5	
3	7	
4	13	
5	15	
6	5	
7	4	
8	27	
Total:	80	

1. (4 points) Which of the following are "indeterminate forms"? (*Hint:* there are 4.)
   - ☐ $1/0$
   - ☐ $0/\infty$
   - ☐ $0/0$
   - ☐ $\infty/0$
   - ☐ ∞/∞
   - ☐ $0 \cdot \infty$
   - ☐ 0^{∞}
   - ☐ $\infty - \infty$

2. (5 points) Compute the following limit:

$$\lim_{x \to \infty} \frac{x^3}{e^{3x}}$$

3. Consider the function $r(x) = x \ln\left(\frac{x+67}{x}\right)$
   (a) (2 points) Which of the following is the correct form of the limit $\lim_{x\to\infty} r(x)$?
   A. $\infty \cdot \infty$ B. $\infty \cdot 1$ C. $\infty \cdot 0$ D. ∞·DNE
   (b) (5 points) Exactly compute $\lim_{x\to\infty} x \ln\left(\frac{x+67}{x}\right)$. Carefully show your work. Anytimes you use L'Hopital's Rule, clearly indicate where you are using it.

4. Ash wants to approximate the value of $\sqrt[5]{32.5}$, but they forgot their phone at home, so they cannot ask ChatGPT.

   (a) (2 points) What is a function $f(x)$ that will help them to get this approximation?

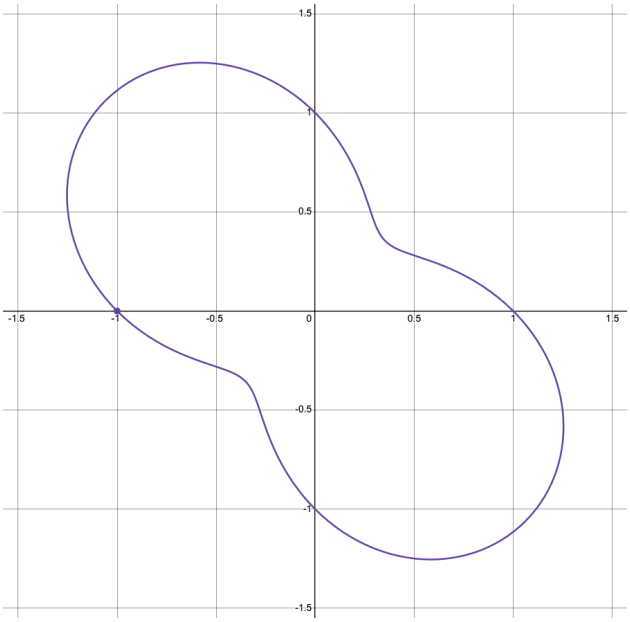
   (b) (2 points) At which point $(a, f(a))$ should Ash draw their tangent line to do the approximation?

   (c) (4 points) Write the equation of the tangent line to your function in (a) at the point you picked in (b).

   (d) (2 points) Use your tangent line above to approximate $\sqrt[5]{32.5}$. **Your final answer should be a simple integer, decimal, or fraction.**

   (e) (3 points) Ash is a curious cat. Is their estimate an overestimate or an underestimate? Use calculus to justify your reasoning. **A graph is not enough justification here. You need an actual computation.**

5. The graph below is given by $x^2 - xy + y^2 = (x^2 + y^2)^{3/2}$ with the point $(-1, 0)$ shown below.

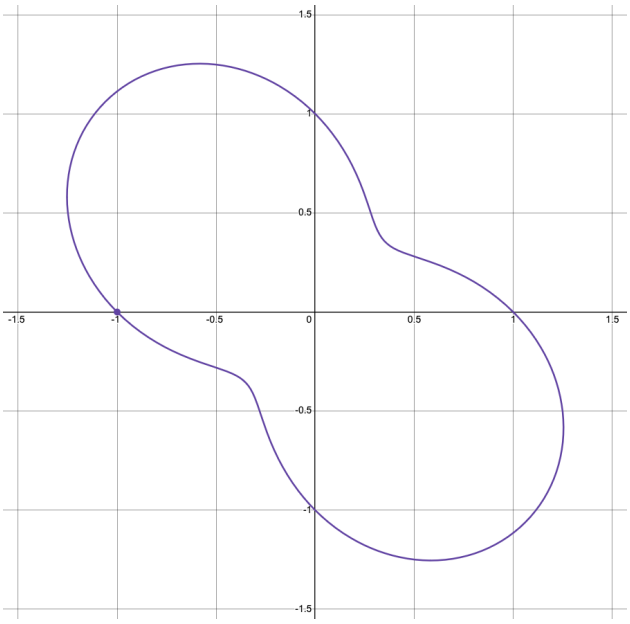


(a) (2 points) What can you say about $\frac{dy}{dx}$ at $(-1, 0)$? A. It is negative. B. It is zero. C. It is positive. D. We cannot say anything.

(b) (6 points) Find $\frac{dy}{dx}$ at the point $(-1, 0)$ for the curve $x^2 - xy + y^2 = (x^2 + y^2)^{3/2}$.

(c) (3 points) Which of the following is the equation of the tangent line to this curve at $(-1, 0)$.

A. $L(x) = -\frac{1}{3}(x + 1)$
B. $L(x) = -\frac{1}{2}(x + 1)$
C. $L(x) = -\frac{4}{5}(x + 1)$
D. $L(x) = -(x + 1)$



(d) (2 points) On the curve at input $x = -1.1$, there are two y values: a y value slightly greater than 0, and a y value slightly greater than 1. Use your answer to part (c) to approximate the y-value closer to 0 when $x = -1.1$. **Your final answer should be a simple fration or a simple decimal.**

(e) (2 points) Which of the following is true for your approximation in (d) using linearization?

A. The approximation is an underestimate as the curve is concave up at $(-1, 0)$.

B. The approximation is an underestimate since the linearization is at -1 and $-1 > -1.1$.

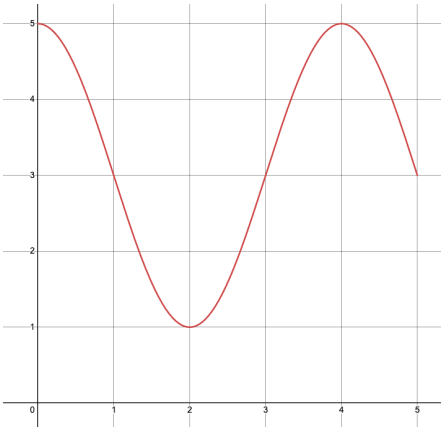
C. The approximation is an overestimate as the curve is concave down at $(-1, 0)$.

D. The approximation is an overerestimate since the linearization is at -1 and $-1.1 < -1$.

6. (5 points) Evaluate the following limit:

$$\lim_{t \to 1} \frac{5t^4 - 4t^2 - 1}{10 - t - 9t^3}$$

7. Consider $f(x)$ whose graph is given below and let ${}_1A_f(x) = \int_1^x f(t)\,\mathrm{d}t$.



(a) (1 point) What can you say about ${}_1A_f(1) = \int_1^1 f(t)\,\mathrm{d}t$.

A. ${}_1A_f(1) < 0$
B. ${}_1A_f(1) = 0$
C. ${}_1A_f(1) > 0$

(b) (1 point) What can you say about ${}_1A_f(0) = \int_1^0 f(t)\,\mathrm{d}t$.

A. ${}_1A_f(0) < 0$
B. ${}_1A_f(0) = 0$
C. ${}_1A_f(0) > 0$

(c) (1 point) On what interval(s) of x in $(0, 5)$ is ${}_1A_f(x) = \int_1^x f(t)\,\mathrm{d}t$ increasing?

A. $(0, 2)$ and $(4, 5)$ only
B. $(2, 4)$ only
C. $(0, 5)$

(d) (1 point) On what interval(s) of x in $(0, 5)$ is ${}_1A_f(x) = \int_1^x f(t)\,\mathrm{d}t$ concave up?

A. $(0, 2)$ and $(4, 5)$ only
B. $(2, 4)$ only
C. $(0, 5)$

(e) Sketch the graph of ${}_1A_f(x)$ on $[0, 5]$ on the axes below, and **label one x-intercept** with exact values.

8. This problem will guide you through the steps to sketch the graph of the following function:

$$h(x) = \frac{x^2 - x - 2}{x^2 - x - 6}.$$

(a) (2 points) What is the domain of $h(x)$?

(b) (2 points) Find the y-intercept of $h(x)$

(c) (2 points) Find the x-intercept of $h(x)$, if there is one.

(d) (2 points) Does $h(x)$ have any discontinuties? If so, what kind and where?

(e) (2 points) What are the horizontal asymptotes of $h(x)$?
Workspace:

- ☐ Only $h(x) = \infty$
- ☐ Only $h(x) = 1$
- ☐ Both $h(x) = 0$ and $h(x) = \infty$
- ☐ This function has no horizontal asymptotes.

(f) (2 points) Make a sign chart of the **first derivative**. What does it tell you about h?

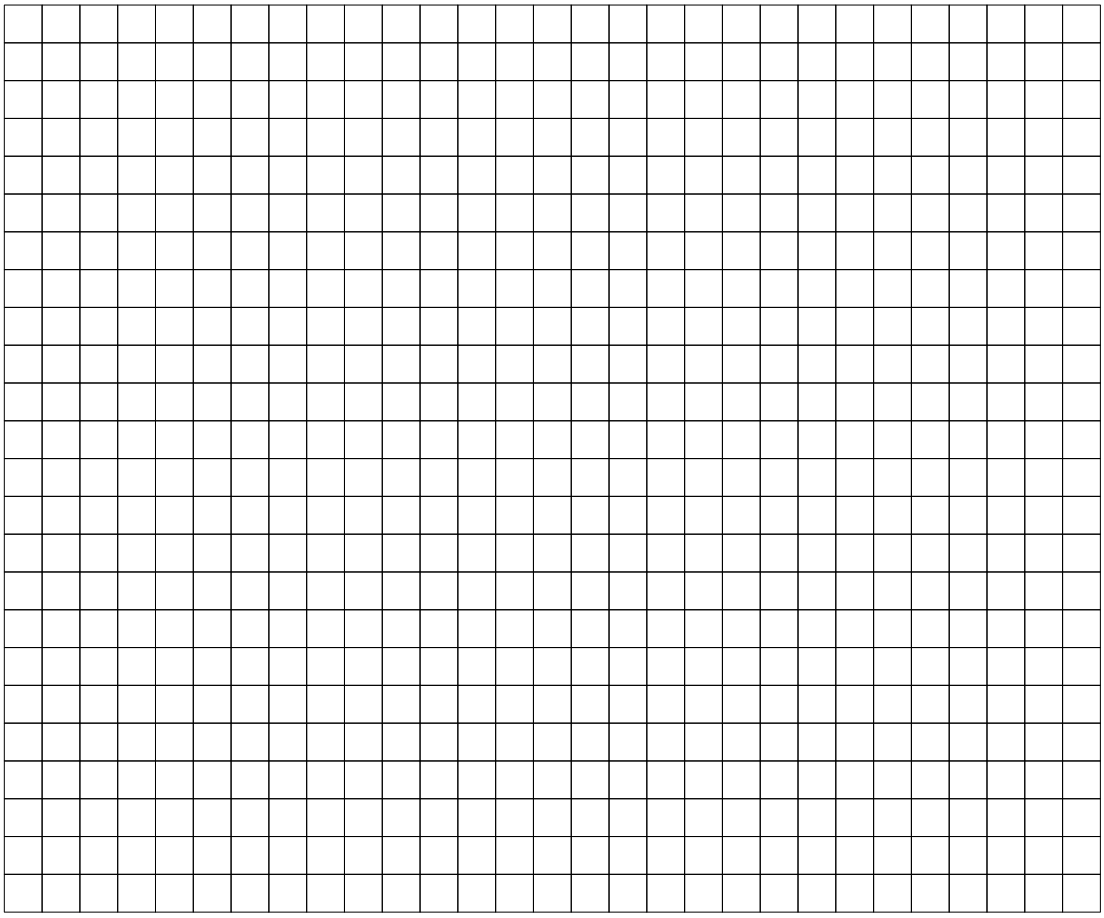
(g) (1 point) Find the point $(x, h(x))$ that is a *local minimum*.

(h) (1 point) Find the point $(x, h(x))$ that is a *local maximum*.

(i) (3 points) Make a sign chart of the **second derivative**. What does it tell you about h? Are there any points of inflection?

(j) (10 points) Using the information from the previous parts, sketch the graph of the function:

$$h(x) = \frac{x^2 - x - 2}{x^2 - x - 6}$$



End of exam.