Emory University
Final Exam Review: L'Hopitals, Linear Approximation, and Curve Sketching
MA-111 Calculus I
Date: December 15, 2025
Instructor: Prof. Mitchell Scott

Student ID ____________ Name: ________________________

Please read the following instructions carefully.

- You have 90 minutes to complete this exam. This question booklet contains 8 questions, 8 pages (including the cover) for the total of 67 points/marks. Check to see if any pages are missing. DO NOT scribble or do rough work or make any stray marks on it. Use separate sheet for rough work.
- All the questions are compulsory and all the notations have their usual meaning. You can use standard results directly by writing its statement appropriately. **Read the instructions for individual questions carefully** before answering the questions.
- More instructions here.

Question	Points	Score
1	4	
2	5	
3	7	
4	5	
5	0	
6	15	
7	4	
8	27	
Total:	67	

1. (4 points) Which of the following are "indeterminate forms"? (*Hint:* there are 4.)

what is an indeterminate form?

- ☐ $1/0$ $1/x$
- ☐ $0/\infty$
- ☑ $0/0$ ✱
- ☐ $\infty/0$
- ☑ $\pm\infty/\pm\infty$ ✱
- ☑ $0(\pm\infty)$
- ☐ 0^∞
- ☑ $\infty-\infty$

$\lim_{x\to\infty} \frac{1}{x}\cdot x = 1$

$\lim_{x\to\infty} \frac{1}{x^2}x = \lim_{x\to\infty}\frac{1}{x}$

$\lim_{x\to\infty} \frac{1}{x}x^2 = \infty$

$\lim_{x\to\infty} x^2 - x \to \infty$

$\lim_{x\to\infty} x - x^2 \to -\infty$

$\lim_{x\to\infty} x - x \to 0$

2. (5 points) Compute the following limit:

$$\lim_{x\to\infty}\frac{x^3}{e^{3x}} = \frac{\infty}{\infty}$$

e^x grows faster than x^p

$$\lim_{x\to\infty}\frac{x^3}{e^{3x}} \overset{L'H}{=} \lim_{x\to\infty}\frac{3x^2}{e^{3x}\cdot 3} = \frac{\infty}{\infty} \overset{L'H}{=} \lim_{x\to\infty}\frac{6x}{9e^{3x}} = \frac{\infty}{\infty} \overset{L'H}{=} \lim_{x\to\infty}\frac{6}{27e^{3x}} = \frac{6}{\infty} = 0$$

3. Consider the function $r(x) = x\ln\left(\frac{x+67}{x}\right)$ $\quad \infty\cdot \lim_{x\to\infty}\ln\left(\frac{x+67}{x}\right) = \lim_{x\to\infty}\ln\left(1+\frac{67}{x}\right) = \ln(1) = 0$

   (a) (2 points) Which of the following is the correct form of the limit $\lim_{x\to\infty} r(x)$?
   A. $\infty\cdot\infty$ B. $\infty\cdot 1$ C. $\infty\cdot 0$ D. $\infty\cdot$DNE

   (b) (5 points) Exactly compute $\lim_{x\to\infty} x\ln\left(\frac{x+67}{x}\right)$. Carefully show your work. Anytimes you use L'Hopital's Rule, clearly indicate where you are using it.

$$\lim_{x\to\infty} x\ln\left(\frac{x+67}{x}\right) = \lim_{x\to\infty}\frac{\ln\left(\frac{x+67}{x}\right)}{1/x} = \frac{0}{0} \overset{L'H}{=} \lim_{x\to\infty}\frac{\frac{1}{1+67/x}\cdot(-67\cancel{x^{-2}})}{-\cancel{x^{-2}}} = \frac{+67}{1+\frac{67}{x}} = \boxed{\frac{67}{1+0}}$$

$$\lim_{x\to\infty}\frac{x}{\frac{1}{\ln\left(1+\frac{67}{x}\right)}} = \frac{\infty}{\infty} \overset{L'H}{=} \lim_{x\to\infty}\frac{-67x^{-2}}{\left(1+\frac{67}{x}\right)\left[\ln\left(1+\frac{67}{x}\right)\right]^2} = \frac{1}{\infty\cdot 0}$$

$$\frac{d}{dx}\ln(1+67/x)^{-1} = -1\ln(1+67/x)^{-2}\frac{d}{dx}\ln\left(1+67/x\right) = \frac{-1}{\left[\ln\left(1+\frac{67}{x}\right)\right]^2}\cdot\frac{1}{1+67/x}\cdot(-67x^{-2})$$

4. (5 points) Evaluate the following limit ~~integral~~.

$$\lim_{t\to 1} \frac{5t^4 - 4t^2 - 1}{10 - t - 9t^3} = \frac{5-4-1}{10-1-9} = \frac{0}{0}$$

$$\lim_{t\to 1} \frac{5t^4-4t^2-1}{10-t-9t^3} \overset{L'H}{=} \lim_{t\to 1} \frac{20t^3-8t}{-1-27t^2} = \frac{20-8}{-1-27} = \frac{12}{-28} = \boxed{-\frac{3}{7}}$$

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5. Determine the linear approximation for $f(x) = \sqrt[5]{x}$ at $x = 32$. Use this to approximate the value of $\sqrt[5]{32.5}$.

$32.5^{1/5}$ $\quad x^{1/5}$ $\quad 32.5^{x}$

Linear approximation: 1.) Tangent line 2.) point-slope form

$f(x) = x^{1/5}$ $\quad f'(x) = \frac{1}{5}x^{-4/5} = \frac{1}{5x^{4/5}}$ $\quad a=2$ since $32^{1/5}=2$

$2\cdot 2\cdot 2\cdot 2\cdot 2 = 32$

$f'(32) = \frac{1}{5(32)^{4/5}} = \frac{1}{5(\sqrt[5]{32})^4} = \frac{1}{5\cdot 2^4} = \frac{1}{5\cdot 16}$ $\quad f(32)=2$

$L(x) = \frac{1}{5\cdot 16}(x-32)+2$

$L(32.5) = \frac{1}{5\cdot 16}(32.5-32)+2 = \frac{1}{2\cdot 5\cdot 16}+2 = 2+\frac{1}{160}$

$= 2.00625$

$f''(x) = \frac{-4}{25x^{9/5}} < 0$ $\quad f(32.5) = 2.006211\ldots$

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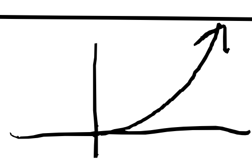
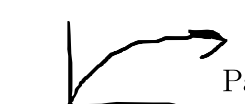
x^p if $p>1$



concave up, tangent is under

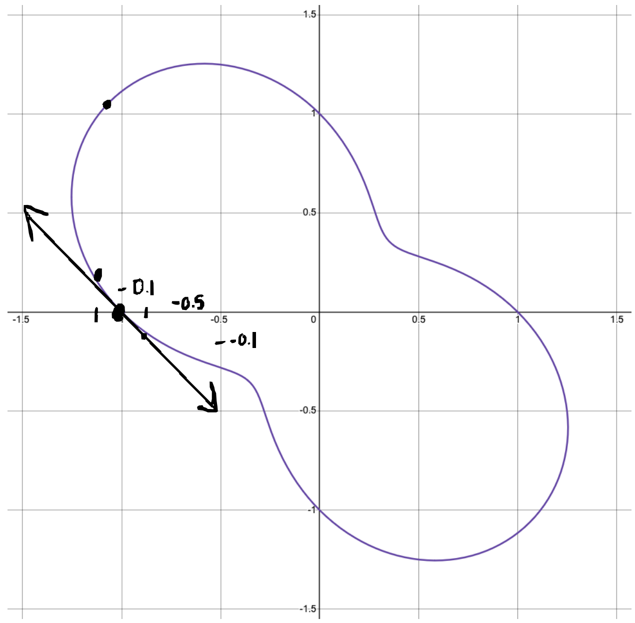
if $p<1$



concave down, tangent is over

$y = y(x)$

6. The graph below is given by $x^2 - xy + y^2 = (x^2+y^2)^{3/2}$ with the point $(-1, 0)$ shown below.



(a) (2 points) What can you say about $\frac{dy}{dx}$ at $(-1, 0)$? A. It is negative. B. It is zero. C. It is positive. D. We cannot say anything.

(b) (6 points) Find $\frac{dy}{dx}$ at the point $(-1, 0)$ for the curve $x^2 - xy + y^2 = (x^2+y^2)^{3/2}$.

$$\frac{d}{dx}\left(x^2 - xy + y^2\right) = \frac{d}{dx}\left(x^2+y^2\right)^{3/2} \qquad (x,y) = (-1,0)$$

$$2x - \left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} = \frac{3}{2}\left(x^2+y^2\right)^{1/2}\cdot\left(2x + 2y\frac{dy}{dx}\right)$$

$$(-1)2 + \left(+\frac{dy}{dx} + 0\right) + 0 = \frac{3}{2}\left((-1)^2 + 0^2\right)^{1/2}\left((-1)2 + 0\right)$$

$$\frac{dy}{dx} = 2 + \frac{3}{2}(1)^{1/2}(-2) = 2 + (-3) = \boxed{-1 = \frac{dy}{dx}}$$

(c) (3 points) Which of the following is the equation of the tangent line to this curve at $(-1, 0)$.

A. $L(x) = -\frac{1}{3}(x+1)$
B. $L(x) = -\frac{1}{2}(x+1)$
C. $L(x) = -\frac{4}{5}(x+1)$
D. $L(x) = -(x+1)$

$$L(x) - y_1 = m(x - x_1)$$

$$L(x) = -1\left(x + (+1)\right) = -(x+1)$$

$$= -x - 1$$

(d) (2 points) On the curve at input $x = -1.1$, there are two y values: a y value slightly greater than 0, and a y value slightly greater than 1. Use your answer to part (c) to approximate the y-value closer to 0 when $x = -1.1$. **Your final answer should be a simple fration or a simple decimal.**

$L(x) = -x - 1$ so $L(-1.1) = +(+1.1) - 1 = 1.1 - 1 = 0.1$

(e) (2 points) Which of the following is true for your approximation in (d) using linearization?

A. The approximation is an ~~over~~ under estimate as the curve is concave up at $(-1, 0)$.

B. The approximation is an ~~over~~ under estimate since the linearization is at -1 and $-1 > -1.1$.

C. The approximation is an ~~under~~ over estimate as the curve is ~~concave~~ down at $(-1, 0)$.

D. The approximation is an ~~under~~ over estimate since the linearization is at 1 and $-1.1 < -1$.

7. Consider $f(x)$ whose graph is given below and let ${}_1A_f(x) = \int_1^x f(t)\,dt$.

(a) (1 point) What can you say about ${}_1A_f(1) = \int_1^1 f(t)\,dt$.

A. ${}_1A_f(1) < 0$

B. ${}_1A_f(1) = 0$

C. ${}_1A_f(1) > 0$

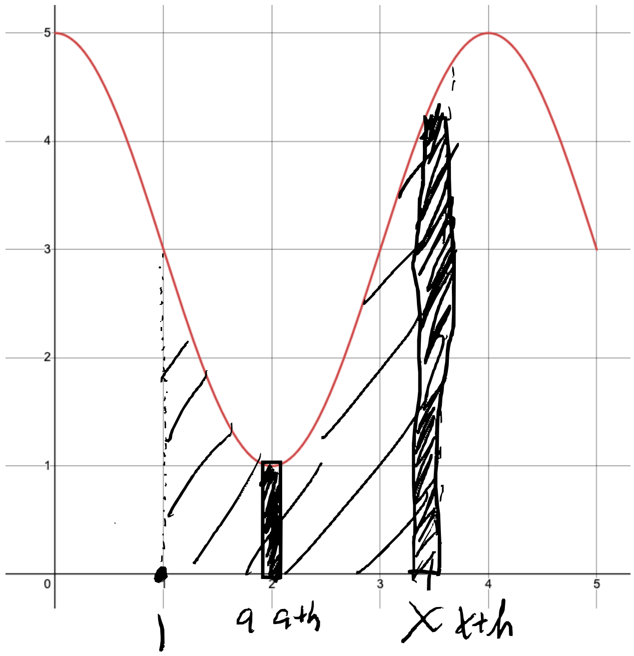
(b) (1 point) What can you say about ${}_1A_f(0) = \int_1^0 f(t)\,dt$. $= -\int_0^1 f(t)dt$

A. ${}_1A_f(0) < 0$

B. ${}_1A_f(0) = 0$

$_1A_f(1) = \int_1^1 f(t)\,dt$

$_1A_f(0) = -\int_0^1 f(t)\,dt$
$= -$ signed area



C. $_1A_f(0) > 0$

(c) (1 point) On what interval(s) of x in $(0, 5)$ is $_1A_f(x) = \int_1^x f(t)\,dt$ increasing?

A. $(0, 2)$ and $(4, 5)$ only

B. $(2, 4)$ only

C. $(0, 5)$

(d) (1 point) On what interval(s) of x in $(0, 5)$ is $_1A_f(x) = \int_1^x f(t)\,dt$ concave up?

~~A.~~ $(0, 2)$ and $(4, 5)$ only

B. $(2, 4)$ only

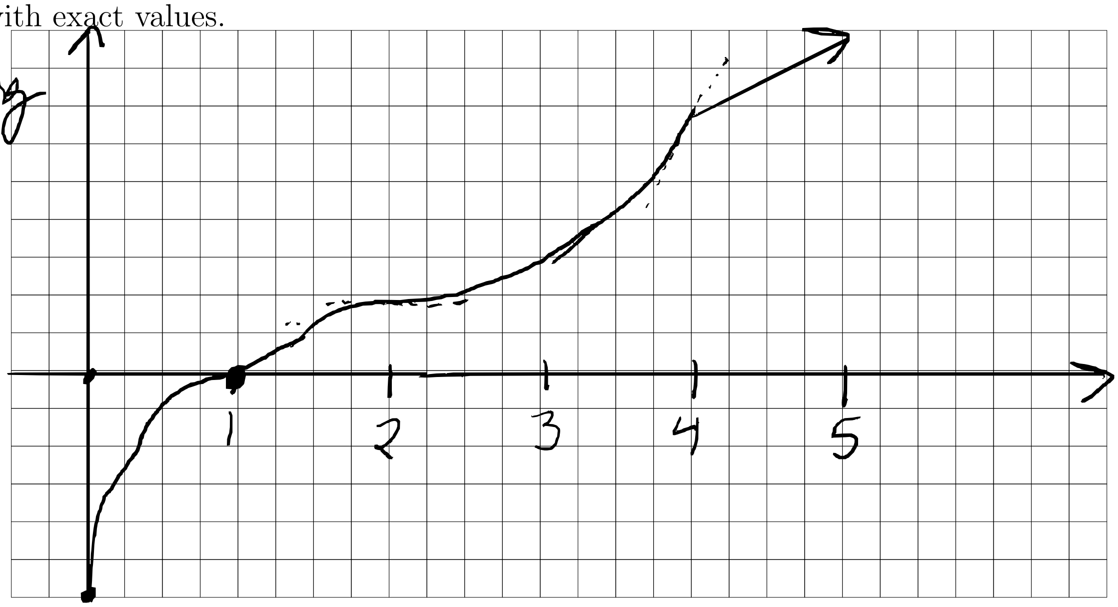
~~C.~~ $(0, 5)$

(e) Sketch the graph of $_1A_f(x)$ on $[0, 5]$ on the axes below, and **label one x-intercept** with exact values.

always increasing
y int x int

$x = 1$

$_1A_f(1) = 0$



8. This problem will guide you through the steps to sketch the graph of the following function:

$$h(x) = \frac{x^2 - x - 2}{x^2 - x - 6}.$$

(a) (2 points) What is the domain of $h(x)$?

all real numbers except when $x^2-x-6=0 \Rightarrow (x-3)(x+2)=0$

if $x=3, -2$

(b) (2 points) Find the y-intercept of $h(x)$ y intercept is when $x=0$

$h(0) = \frac{0-0-2}{0-0-6} = \frac{-2}{-6} = \frac{1}{3}$ $(0, \frac{1}{3})$

(c) (2 points) Find the x-intercept of $h(x)$, if there is one.

when $h(x)=0$ $(x^2-x-2)=0 \Rightarrow (x-2)(x+1)=0$

$(2,0)$ and $(-1,0)$

(d) (2 points) Does $h(x)$ have any discontinuties? If so, what kind and where?

$\lim_{x\to 3^+} \frac{(x-2)(x+1)}{(x+2)(x-3)} = \frac{(3-2)(3+1)}{(5)\,0^+} = +\infty$

-2 is vertical asymptote $\uparrow\downarrow$ 3 is vertical asymptote $\searrow\nwarrow$

(e) (2 points) What are the horizontal asymptotes of $h(x)$?
Workspace:

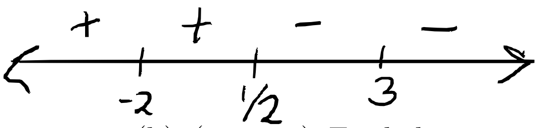
$\lim_{x\to\infty} \frac{1x^2-x-2}{1x^2-x-6} = 1$ $\lim_{x\to-\infty} \frac{x^2-x-2}{x^2-x-6} = 1$

- ☐ Only $h(x) = \infty$
- ☑ Only $h(x) = 1$
- ☐ Both $h(x) = 0$ and $h(x) = \infty$
- ☐ This function has no horizontal asymptotes.

(f) (2 points) Make a sign chart of the **first derivative**. What does it tell you about h?

$h'(x) = \frac{(x^2-x-6)(2x-1) - (x^2-x-2)(2x-1)}{(x^2-x-6)^2} = \frac{-8x+4}{(x+2)^2(x-3)^2}$ $x = \frac{1}{2}$

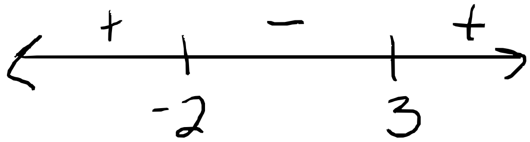
(g) (1 point) Find the point $(x, h(x))$ that is a *local minimum.*



(h) (1 point) Find the point $(x, h(x))$ that is a *local maximum.*

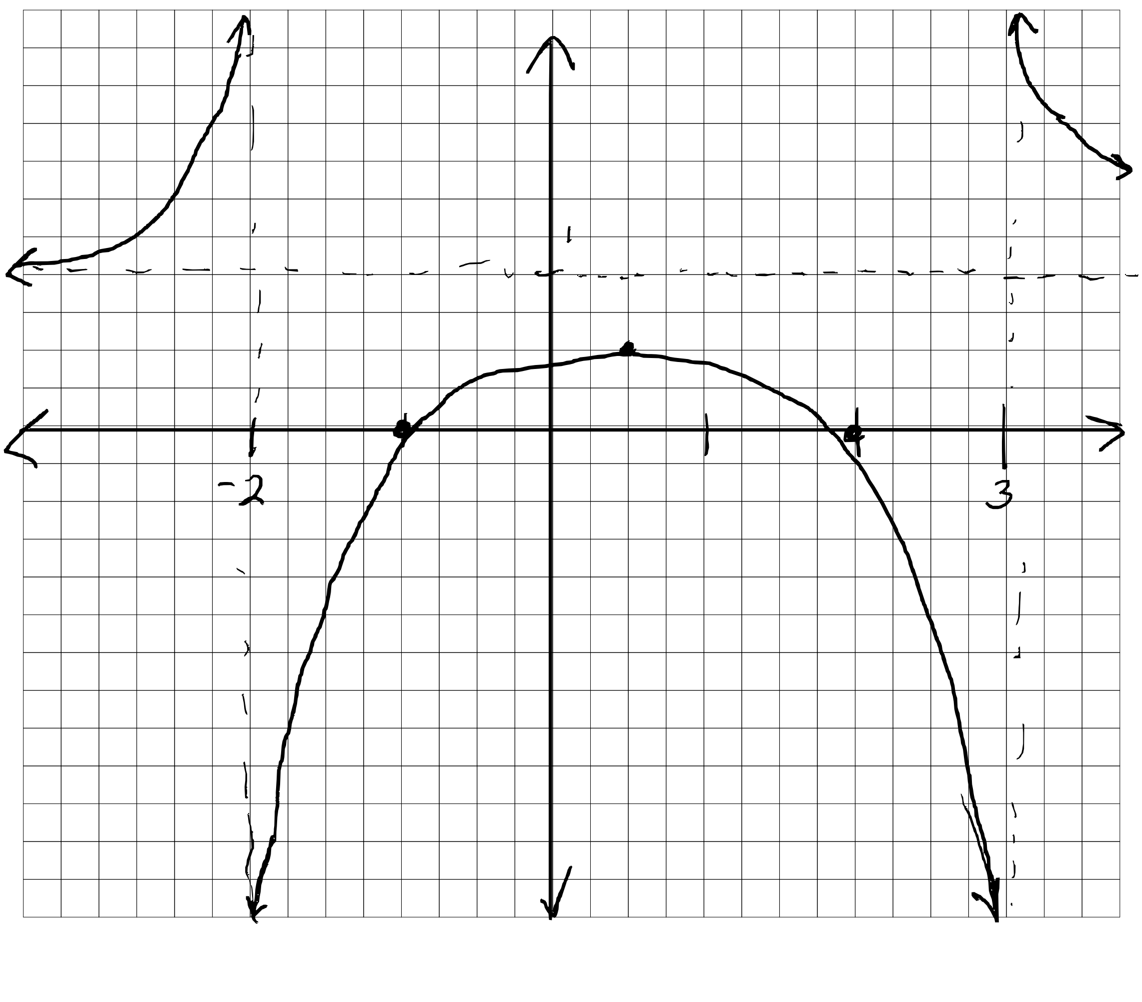
$x = 1/2$

(i) (3 points) Make a sign chart of the **second derivative**. What does it tell you about h? Are there any points of inflection?



(j) (10 points) Using the information from the previous parts, sketch the graph of the function:

$$h(x) = \frac{x^2 - x - 2}{x^2 - x - 6}$$



End of exam.