Emory University
Exam #1
MA-111 Calculus I
Date: December 10, 2025
Instructor: Prof. Mitchell Scott

Student ID ____________ Name: __________________________

Please read the following instructions carefully.

- **This is simply a review session!** This question booklet contains 3 questions, 6 pages (including the cover) for the total of 34 points/marks. Check to see if any pages are missing. DO NOT scribble or do rough work or make any stray marks on it. Use separate sheet for rough work.
- This is meant to represent what an actual exam might look like. **Read the instructions for individual questions carefully** before answering the questions.
- This was not overseen by the coordinator nor other instructors.

Question	Points	Score
1	12	
2	12	
3	10	
Total:	34	

1. Solve the following definite integrals:

   (a) (4 points)

$$\int_0^4 (-2.125x + 21)\,\mathrm{d}x$$

$-2.125(4) + 21 = 12.5$

$\int_0^4 -2.125x^1 + 21x^0\,dx$

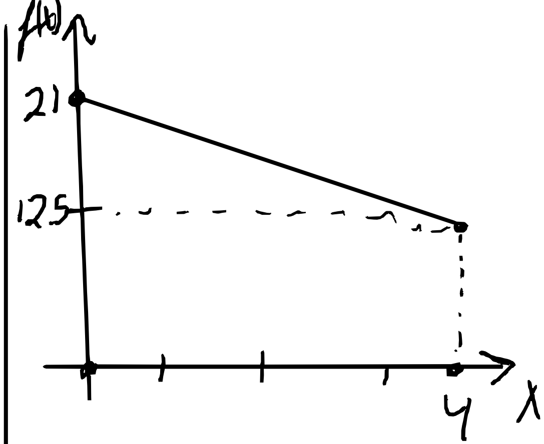
$-2.125\frac{x^2}{2} + 21x^1 \Big|_0^4$

$-2.125\frac{(4)^2}{2} + 21(4) - \left[-2.125\frac{0^2}{2} + 21(0)\right]$

$-2.125(8) + 84 - 0 = -17 + 84$

$= 67$

$\frac{d}{dx} -2.125\frac{x^2}{2} + 21x = -2.125\frac{(2x)}{2} + 21$



$\left(\frac{21 + 12.5}{2}\right)(4-0) = 16.75 \cdot 4$

$= 67$

   (b) (4 points)

$$\int_{x=0}^{x=\ln(1+\pi)} e^x\cos(1-e^x)\,\mathrm{d}x$$

No product or quotient for integrals!

composite function:

$f(g(x)) = \cos(1-e^x)$

$f(x) = \cos(x)$

$g(x) = 1-e^x$

$u = g(x) = 1 - e^x$

$du = 0 - e^x dx \Rightarrow -du = e^x dx$

$u(\ln(1+\pi)) = 1 - e^{\ln(1+\pi)} = 1-(1+\pi) = -\pi$

$u(0) = 1 - e^0 = 0$

$\int_{u=0}^{u=-\pi} \cos(u)(-du) = -\int_0^{-\pi} \cos u\, du = -\sin(u)\Big|_0^{-\pi}$

$= -\sin(-\pi) - (-\sin(0)) = -0 + +0 = 0$

$$\int_0^{\ln(1+\pi)} e^x \cos(1-e^x)\,dx$$

$$u = e^x \qquad du = e^x dx$$

$$u(\ln(1+\pi)) = e^{\ln(1+\pi)} = 1+\pi$$

$$u(0) = e^0 = 1$$

$$\int_1^{1+\pi} \cos(1-u)\,du$$

$$w = 1-u \qquad dw = -du \Rightarrow -dw = du$$

$$w(1+\pi)\quad 1+(-1-\pi) = -\pi$$

$$w(1) = 1-1 = 0$$

$$\int_0^{-\pi} \cos(w)(-dw) = -\int_0^{\pi} \cos(w)\,dw = -\sin(w)\Big|_0^{-\pi}$$

$$= 0$$

(c) (4 points)

$$\int_{x=-\pi}^{x=\frac{\pi}{2}} \cos(x)\cos(\sin(x))\,\mathrm{d}x$$

$u = \sin(x)$

$du = \cos(x)dx$

$$\int_{x=-\pi}^{x=\frac{\pi}{2}} \cos(u)\,du = \sin(u)\Big|_{x=-\pi}^{x=\frac{\pi}{2}}$$

$u(\frac{\pi}{2}) = \sin(\frac{\pi}{2}) = 1 \quad u(-\pi) = \sin(-\pi) = 0$

$$\int_0^1 \cos(u)\,du = \sin(u)\Big|_0^1$$

$$= \sin(1) - \sin(0)$$

$$= \sin(1)$$

$$= \sin(\sin(x))\Big|_{-\pi}^{\pi/2}$$

$$= \sin(\sin(\tfrac{\pi}{2})) - \sin(\sin(-\pi))$$

$$= \sin(1) - \sin(0) = \boxed{\sin(1)}$$

2. Solve the following indefinite integrals:

(a) (4 points)

$$\int \frac{\arcsin(x)^1}{\sqrt{1-x^2}}\,\mathrm{d}x$$

~~$u = 1 - x^2 \quad du = -2x\,dx$~~

$u = \arcsin(x)$

$du = \dfrac{1}{\sqrt{1-x^2}}dx$

$$\int u^1 du = \frac{u^{1+1}}{1+1} + C = \frac{u^2}{2} + C$$

$$= \frac{\arcsin(x)^2}{2} + C$$

$$\frac{d}{dx}\frac{\arcsin(x)^2}{2} + C = \frac{2\arcsin(x)}{2}\frac{d}{dx}\arcsin(x)$$

$$= \arcsin(x) \cdot \frac{1}{\sqrt{1-x^2}}$$

(b) (4 points)

$$\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,\mathrm{d}x$$

$u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2} x^{-1/2} dx = \frac{1}{2} \frac{1}{\sqrt{x}} dx$

$= 2du = \frac{1}{\sqrt{x}} dx$

$\int 2\cos(u)\,du = 2\sin(u) + C$

$= 2\sin(\sqrt{x}) + C$

$\frac{d}{dx} 2\sin(\sqrt{x}) + C = 2\cos(\sqrt{x}) \cdot \frac{d}{dx}\sqrt{x} = \frac{\cancel{2}\cos(\sqrt{x})}{\cancel{2}\sqrt{x}} = \frac{\cos(\sqrt{x})}{\sqrt{x}}$

(c) (4 points)

$$\int \left(x\cos\left(x^2+1\right) + \frac{x}{x^2+1}\right)\mathrm{d}x$$

$\int a + b\,dx = \int a\,dx + \int b\,dx$

$u = x^2 + 1$

$du = 2x\,dx \Rightarrow \frac{du}{2} = x\,dx$

$\int \cos(u) \frac{du}{2} + \int \frac{1}{u} \frac{du}{2}$

$\frac{1}{2}\sin(u) + \frac{1}{2}\ln|u| + C$

$\frac{1}{2}\sin(x^2+1) + \frac{1}{2}\ln|x^2+1| + C$

3. Solve the following integrals:

   (a) (4 points)

$$\int \left(x\cos\left(x^2+1\right) + \frac{1}{x^2+1} \right) \mathrm{d}x$$

$u = x^2+1$

$du = 2x\,dx \Rightarrow \frac{du}{2} = x\,dx$

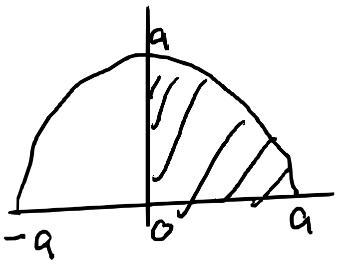
$\int x\cos(x^2+1)\,dx + \int \frac{1}{x^2+1}\,dx$

$\int \frac{1}{2}\cos(u)\,du + \arctan(x) + C$

$\frac{1}{2}\sin(u) + \arctan(x) + C$

$\frac{1}{2}\sin(x^2+1) + \arctan(x) + C$

(b) (6 points) Using the trigonometric identity $\cos^2(x) = \frac{1}{2}(\cos(2x)+1)$, solve



$$\int_0^a \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{4}a^2\pi$$

$\frac{x^2}{a^2} = \left(\frac{x}{a}\right)^2$

$\frac{x}{a} = \sin(t)$

$x = a\sin(t)$

$dx = a\cos(t)dt$

$a = a\sin(t)$, $t = \pi/2$

$0 = a\sin(t)$, $t = 0$

$$\int_0^a \sqrt{a^2\left(1-\frac{x^2}{a^2}\right)}\,dx = \int_0^a a\sqrt{1-x^2/a^2}\,dx$$

$$\int_0^{\pi/2} a^2\sqrt{1-\sin^2(t)}\,\cos(t)dt$$

$1 = \sin^2 t + \cos^2 t$

$\sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = \cos t$

t is between 0 and $\pi/2$

$$\int_0^{\pi/2} a^2\cos^2(t)\,dt$$

$$= \int_0^{\pi/2} a^2\left(\frac{1}{2}\cos(2t)+1\right)dt$$

$$\frac{a^2}{2}\left(\frac{1}{2}\sin(2t)+t\right)\Big|_0^{\pi/2} = \frac{a^2}{2}\left(\frac{1}{2}\sin\left(2\frac{\pi}{2}\right)+\frac{\pi}{2}-\left[\frac{1}{2}\sin(0)+0\right]\right)$$

$$= \frac{a^2}{2}\left(\frac{1}{2}\sin(\pi)+\frac{\pi}{2}-0\right)$$

$$= \frac{a^2}{2}\left(0+\frac{\pi}{2}\right) = \frac{a^2\pi}{4}$$

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