Emory University
Exam # 3 Review Sheet
MA-111 Calculus I
Date: November 14, 2025

Instructor: Mitchell Scott

Student ID ____________ Name: ________________________

Please read the following instructions carefully.

- You have 90 minutes to complete this exam. This question booklet contains 4 questions, 15 pages (including the cover) for the total of 108 points/marks. Check to see if any pages are missing. DO NOT scribble or do rough work or make any stray marks on it. Use separate sheet for rough work.
- All the questions are compulsory and all the notations have their usual meaning. You can use standard results directly by writing its statement appropriately. **Read the instructions for individual questions carefully** before answering the questions.
- More instructions here.

Question	Points	Score
1	27	
2	27	
3	27	
4	27	
Total:	108	

1. This problem will guide you through the steps to sketch the graph of the following function:

$$f(x) = \frac{e^x}{(x+2)^2}.$$

(a) (2 points) What is the domain of $f(x)$?

Solution: Since this is a rational function with familiar functions in the numerator and denominator, we know the only possible points, where the function isn't defined, is when the denominator is zero, so for our case, we have when $(x+2)^2 = 0 \implies x = -2$, there is a point when the function $f(x)$ isn't defined. This means $f(x)$ has a domain of all real numbers except when $x = -2$.

(b) (2 points) Find the y-intercept of $f(x)$

Solution: The y-intercept of a function is the output of the function when $x = 0$. Plugging this in, we have $f(0) = e^0/(0-2)^2 = 1/4$.

(c) (2 points) Find the x-intercept of $f(x)$, if there is one.

Solution: The x-intercept of a function is when $f(x) = 0$. Since this is a rational function, this occurs only when $e^x = 0$. However, we know that there are no values of x where $e^x = 0$, so there are no x-intercepts.

(d) (2 points) Does $f(x)$ have any discontinuities? If so, what kind and where?

Solution: Discontinuities happen when $x \to -2$. From the left, and right respectively, we have

$$\lim_{x\to -2^-} f(x) = \frac{e^{-2}}{(0^-)^2} = \infty, \qquad \lim_{x\to -2^+} f(x) = \frac{e^{-2}}{(0^+)^2} = \infty.$$

By classifying discontinuities, since both limits increase without bound, this is a vertical asymptote.

(e) (2 points) What are the horizontal asymptotes of $f(x)$?

Solution: Before we are able to figure out the horizontal asymptotes, we must compute the limits to infinity

$$\begin{aligned}\lim_{x\to\infty}\frac{e^x}{(x-2)^2} &= \frac{\infty}{\infty}\\ &\overset{\text{L'Hopital}}{=} \lim_{x\to\infty}\frac{e^x}{2(x-2)} = \frac{\infty}{\infty}\\ &\overset{\text{L'Hopital}}{=} \lim_{x\to\infty}\frac{e^x}{2} = \infty.\end{aligned}$$

Similarly, we have

$$\lim_{x\to-\infty}\frac{e^x}{(x-2)^2} = \frac{0}{\infty} = 0$$

Since we don't have finite answers for the limit as $x \to \infty$, this limit increases without bound, then there is no horizontal asymptote. However, as $x \to -\infty$, the limit converges to 0, so that means $y = 0$ is a horizontal asymptote.

- √ **Only** $f(x) = 0$
- ☐ Only $f(x) = \infty$
- ☐ Both $f(x) = 0$ and $f(x) = \infty$
- ☐ This function has no horizontal asymptotes.

(f) (2 points) Make a sign chart of the **first derivative**. What does it tell you about f?

Solution:
We proceed with the quotient rule:

$$f'(x) = \frac{(x+2)^2e^x - 2e^x(x+2)}{\left((x+2)^2\right)^2} = \frac{e^x(x+2)\left((x+2)-2\right)}{(x+2)^4} = \frac{xe^x}{(x+2)^3}$$

The points where f' DNE or $f'(x) = 0$ are when $x = -2$ or $x = 0$, respectively; This means that the sign chart would look like:

$$\begin{array}{cccccc} f & \nearrow & \text{DNE} & \searrow & \rightarrow & \nearrow \\ f' & + & \text{DNE} & - & 0 & + \\ x & \longleftarrow & -2 & \text{———} & 0 & \longrightarrow \end{array}$$

(g) (1 point) Find the point $(x, f(x))$ that is a *local minimum*.

Solution: Looking at the chart above, we see that $f(x)$ has a local minimum when $x = 0$. This is since the derivative is decreasing before $x = 0$ and increasing after.

(h) (1 point) Find the point $(x, f(x))$ that is a *local maximum.*

Solution: We know there can be no local max as there is a vertical asymptote at $x = -2$. Since both $x = -2^-$ and $x = -2^+$ increase without bound, there cannot be a local max.

(i) (3 points) Make a sign chart of the **second derivative**. What does it tell you about f? Are there any points of inflection?

Solution:
We proceed with the quotient rule:

$$\begin{aligned} f''(x) &= \frac{(x+2)^3\,(xe^x + e^x) - 3xe^x(x+2)^2}{\left((x+2)^3\right)^2} \\ &= \frac{(x+2)(xe^x - e^x) - 3xe^x}{(x+2)^4} \\ &= \frac{e^x(x^2+2)}{(x+2)^4} \end{aligned}$$

The points where f'' DNE are when the denominator equals 0, which is when $x = -2$. The points where $f''(x) = 0$ is no point. We know that $e^x \neq 0$ and $x^2 + 2$ can never be zero. This means the only possible point of inflection is $x = -2$.

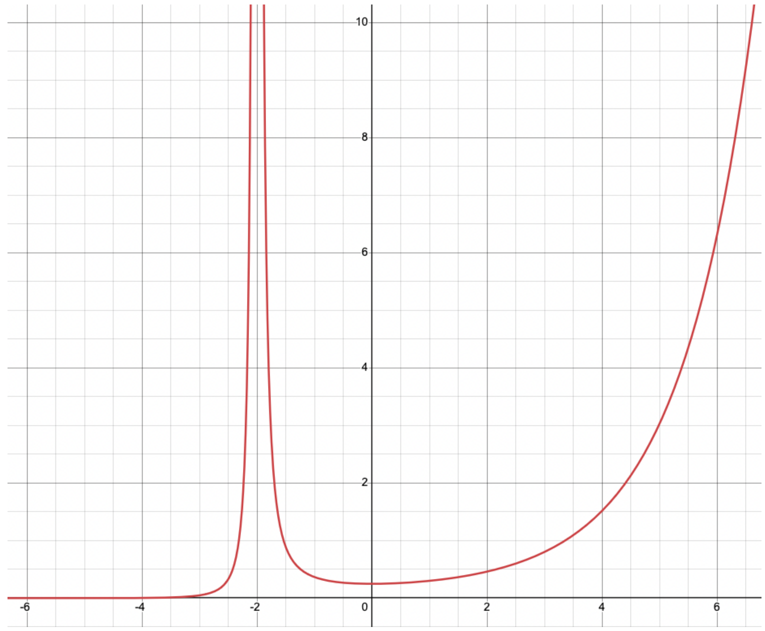
$$\begin{array}{cccc} f & \cup & \text{DNE} & \cup \\ f'' & + & \text{DNE} & + \\ x & \longleftarrow & \underset{-2}{|} & \longrightarrow \end{array}$$

However, when we check the concavity of $f(x)$, we see that it is concave up before $x = -2$ and concave up after $x = -2$, then there is not a point where the concavity changes, so there are no inflection points. In fact, the function f is concave up on the entire domain.

(j) (10 points) Using the information from the previous parts, sketch the graph of the function:

$$f(x) = \frac{e^x}{(x+2)^2}$$

Solution:



2. This problem will guide you through the steps to sketch the graph of the following function:
$$g(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + 3$$
   (a) (2 points) What is the domain of $g(x)$?

   Solution: This is a polynomial, which is a familiar function, so we know that the domain is going to be all real number. This means for any input you give me, I can give you an output.

   (b) (2 points) Find the y-intercept of $g(x)$

   Solution: The y-intercept is when $x = 0$, so for this problem, we have $g(0) = \frac{1}{6}(0)^3 + \frac{1}{2}(0)^2 + 3 = 3$. So the function crosses the y-axis at $(0, 3)$.

   (c) (2 points) Find the x-intercept of $g(x)$, if there is one.

   Solution: There certainly is a x-intercept; however, there is no nice way to factor this polynomial. This means we would **NOT** expect you to be able to find the x-intercept(s). Technically, just like the quadratic formula, there is a cubic formula, but that is again **NOT** expected from you.

(d) (2 points) Does $g(x)$ have any discontinuties? If so, what kind and where?

> **Solution:** No, as we talked about, the function is a polynomial, and is continuous everywhere.

(e) (2 points) What are the horizontal asymptotes of $g(x)$?

> **Solution:** Before we are able to figure out the horizontal asymptotes, we must compute the limits to infinity
>
> $$\lim_{x\to\infty} \frac{1}{6}x^3 + \frac{1}{2}x^2 + 3 = \infty, \qquad \lim_{x\to-\infty} \frac{1}{6}x^3 + \frac{1}{2}x^2 + 3 = -\infty.$$
>
> Since we don't have finite answers, these limits increase (decrease) without bound, then there are no horizontal asymptotes.

- ☐ Only $g(x) = 0$
- ☐ Only $g(x) = \infty$
- ☐ Both $g(x) = 0$ and $g(x) = \infty$
- √ **This function has no horizontal asymptotes.**

(f) (2 points) Make a sign chart of the **first derivative**. What does it tell you about g?

> **Solution:**
> We proceed with the quotient rule:
>
> $$g'(x) = \frac{3}{6}x^2 + \frac{2}{2}x = \frac{1}{2}x^2 + x = x\left(\frac{1}{2}x + 1\right)$$
>
> There are no points where g' DNE. So we need to find the points where $g'(x) = 0$. In the factored form, we see that either $x = 0$ or $x = -2$. This means that the sign chart would look like:

f	$\nearrow$	$\rightarrow$	$\searrow$	$\rightarrow$	$\nearrow$
f'	$+$	0	$-$	0	$+$
x		-2		0	

(g) (1 point) Find the point $(x, g(x))$ that is a *local minimum*.

> **Solution:** Looking at the chart above, we see that $g(x)$ has a local minimum when $x = 0$. This is since the derivative is decreasing before $x = 0$ and increasing after.

(h) (1 point) Find the point $(x, g(x))$ that is a *local maximum*.

> **Solution:** Looking at the chart above, we see that $g(x)$ has a local maximum when $x = -2$. This is since the derivative is increasing before $x = -2$ and decreasing after.

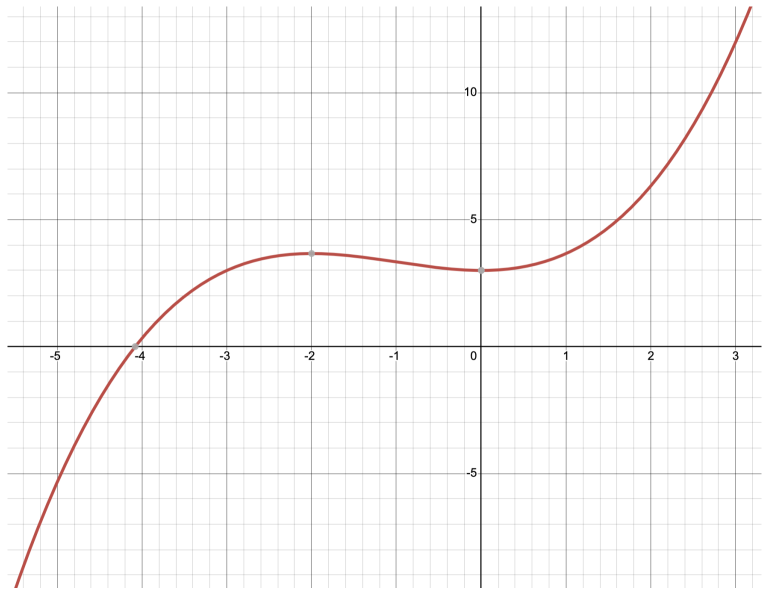
(i) (3 points) Make a sign chart of the **second derivative**. What does it tell you about g? Are there any points of inflection?

> **Solution:**
>
> $$g''(x) = \frac{2}{2}x + 1 = x + 1$$
>
> There are no points where g'' DNE. The points where $g''(x) = 0$ is when $x = -1$. This means the only possible point of inflection is $x = -1$. We have to look at the sign chart to figure out if we are right though.
>
> $$\begin{array}{c|ccc} g & \cap & & \cup \\ g'' & - & 0 & + \\ x & & -1 & \end{array}$$
>
> When we check the concavity of $g(x)$, we see that it is concave down before $x = -1$ and concave up after $x = -1$. Since the function changes concavity at $x = -1$, this means that $x = -1$ is an inflection point.

(j) (10 points) Using the information from the previous parts, sketch the graph of the function:

$$g(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + 3$$

> **Solution:**



3. This problem will guide you through the steps to sketch the graph of the following function:

$$h(x) = \frac{x^2 - x - 2}{x^2 - x - 6}.$$

(a) (2 points) What is the domain of $h(x)$?

Solution: Since this is a rational function with familiar functions in the numerator and denominator, we know the only possible points, where the function isn't defined, is when the denominator is zero, so for our case, we can factor the bottom into $x^2 - x - 6 = (x-3)(x+2)$, we have when $(x+2) = 0 \implies x = -2$, and when $(x-3) = 0 \implies x = 3$. This means $f(x)$ has a domain of all real numbers except when $x = -2, 3$.

(b) (2 points) Find the y-intercept of $h(x)$

Solution: The y-intercept of a function is the output of the function when $x = 0$. Plugging this in, we have

$$f(0) = \frac{(0)^2 - (0) - 2}{(0)^2 - (0) - 6} = \frac{-2}{-6} = \frac{1}{3}$$

.

(c) (2 points) Find the x-intercept of $h(x)$, if there is one.

Solution: The x-intercept of a function is when $h(x) = 0$. Since this is a rational function, this occurs only when $x^2 - x - 2 = (x-2)(x+1) = 0$. This means the x intercepts are $x = -1, 2$

(d) (2 points) Does $h(x)$ have any discontinuities? If so, what kind and where?

Solution: Discontinuities happen when there are issues with the denominator, such as being 0. This means we will have discontinuities when $x = -2, 3$. We will use limits to determine what kind of discontinuities we have.

$$\lim_{x\to-2^-} h(x) = \frac{(-2)^2 - (^-2) - 2}{((-2)-3)\,(0^-)} = \infty, \quad \lim_{x\to-2^+} h(x) = \frac{(-2)^2 - (^-2) - 2}{((-2)-3)\,(0^+)} = -\infty.$$

By classifying discontinuities, since both limits increase (decrease) without bound, this is a vertical asymptote. Similarly,

$$\lim_{x\to3^-} h(x) = \frac{(3)^2 - (3) - 2}{((3)+2)\,(0^-)} = -\infty, \quad \lim_{x\to3^+} h(x) = \frac{(3)^2 - (3) - 2}{((3)+2)\,(0^+)} = +\infty.$$

By classifying discontinuities, since both limits decrease (inccrease) without bound, this is a vertical asymptote.

(e) (2 points) What are the horizontal asymptotes of $h(x)$?

Solution: Before we are able to figure out the horizontal asymptotes, we must compute the limits to infinity

$$\begin{aligned}\lim_{x\to\infty} \frac{x^2 - x - 2}{x^2 - x - 6} &= \frac{\infty}{\infty} \\ &\overset{\text{L'Hopital}}{=} \lim_{x\to\infty} \frac{2x-1}{2x-1} = \frac{\infty}{\infty} \\ &\overset{\text{L'Hopital}}{=} \lim_{x\to\infty} \frac{2}{2} = 1.\end{aligned}$$

Similarly, we have

$$\begin{aligned}\lim_{x\to-\infty} \frac{x^2 - x - 2}{x^2 - x - 6} &= \frac{\infty}{\infty} \\ &\overset{\text{L'Hopital}}{=} \lim_{x\to-\infty} \frac{2x-1}{2x-1} = \frac{-\infty}{-\infty} \\ &\overset{\text{L'Hopital}}{=} \lim_{x\to-\infty} \frac{2}{2} = 1.\end{aligned}$$

This means that the two horizontal asymptotes happen to be the same so the horizontal asymptotes are $y = 1$, both going to the left and the right.

- ☐ Only $h(x) = \infty$
- $\surd$ **Only** $h(x) = 1$
- ☐ Both $h(x) = 0$ and $h(x) = \infty$
- ☐ This function has no horizontal asymptotes.

(f) (2 points) Make a sign chart of the **first derivative**. What does it tell you about h?

Solution:
We proceed with the quotient rule:

$$h'(x) = \frac{(x^2 - x - 6)(2x - 1) - (x^2 - x - 2)(2x - 1)}{(x^2 - x - 6)^2} = \frac{-8x + 4}{(x - 3)^2(x + 2)^2}$$

The points where f' DNE or $h'(x) = 0$ are when $x = -2$ or $x = 0$, respectively; This means that the sign chart would look like:

h	$\nearrow$	DNE	$\nearrow$	$\rightarrow$	$\searrow$	DNE	$\searrow$
h'	$+$	DNE	$+$	0	$-$	DNE	$-$
x		-2		$1/2$		3	

(g) (1 point) Find the point $(x, h(x))$ that is a *local minimum*.

Solution: We know there can be no local min as there is a vertical asymptote at $x = -2$ and $x = 3$. Since both $x = -2^+$ and $x = 3^-$ deccrease without bound, there cannot be a local min.

(h) (1 point) Find the point $(x, h(x))$ that is a *local maximum*.

Solution: Looking at the chart above, we see that $h(x)$ has a local maximum when $x = 1/2$. This is since the derivative is increasing before $x = 0$ and decreasing after.

(i) (3 points) Make a sign chart of the **second derivative**. What does it tell you about h? Are there any points of inflection?

Solution:

We proceed with the quotient rule:

$$\begin{aligned} h''(x) &= \frac{(x-3)^2(x+2)^2(-8) - (-8x+4)(2(x^2-x-6)(2x-1))}{(x-3)^4(x+2)^4} \\ &= \frac{-8(x-3)(x+2) + 32x^2 - 16x + 8}{(x-3)^3(x+2)^3} \\ &= \frac{24x^2 - 24x + 56}{(x-3)^3(x+2)^3} \end{aligned}$$

The points where f'' DNE are when the denominator equals 0, which is when $x = -2, 3$. The points where $h''(x) = 0$ is when $3x^2 - 3x + 7 = 0$. However, if we use the quadratic formula, you will find the only roots are imaginary since $\sqrt{b^2 - 4ac} = \sqrt{9 - 4(3)(7)} = \sqrt{-75}$. We put this in the sign chart below.

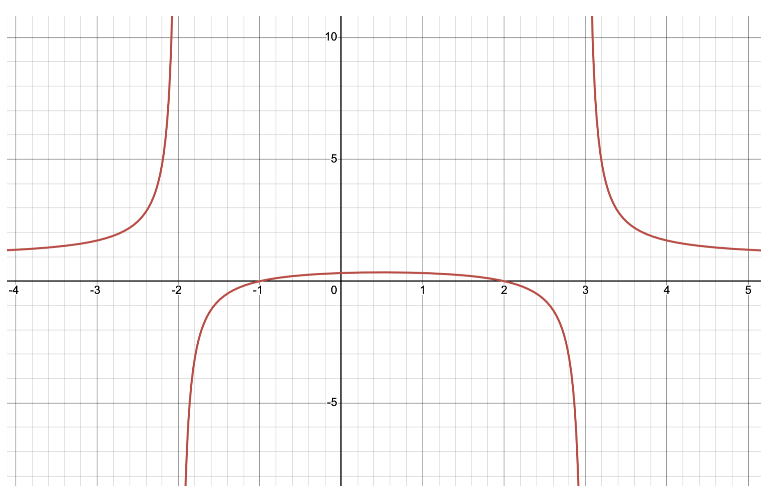
h	$\cup$	DNE	$\cap$	DNE	$\cup$
h''	$+$	DNE	$-$	DNE	$+$
x		-2		3	

We see that h is concave up when $x < -2$, concave down between $-2 < x < 3$, and concave up again when $x > 3$. Even though concavity changes at $x = -2, 3$, since these points aren't in the domain of h, they are not considered inflection points.

(j) (10 points) Using the information from the previous parts, sketch the graph of the function:

$$h(x) = \frac{x^2 - x - 2}{x^2 - x - 6}$$

Solution:



4. This problem will guide you through the steps to sketch the graph of the following function:

$$j(t) = \frac{t^3 - t^2 + 1}{e^t}$$

(a) (2 points) What is the domain of $j(t)$?

> **Solution:** Since this is a rational function with familiar functions in the numerator and denominator, we know the only possible points, where the function isn't defined, is when the denominator is zero, so for our case, we have when $e^t = 0$; however, this never happens. This means $j(t)$ has a domain of all real numbers.

(b) (2 points) Find the y-intercept of $j(t)$

> **Solution:** The y-intercept of a function is the output of the function when $t = 0$. Plugging this in, we have $j(0) = ((0)^3 - (0)^2 + 1)/e^0 = 1$.

(c) (2 points) Find the t-intercept of $j(t)$, if there is one.

> **Solution:** The j-intercept of a function is when $j(t) = 0$. Since this is a rational function, this occurs only when $t^3 - t^2 + 1 = 0$. There certainly is a t-intercept; however, there is no nice way to factor this polynomial. This means we would **NOT** expect you to be able to find the t-intercept(s). Technically, just like the quadratic formula, there is a cubic formula, but that is again **NOT** expected from you.

(d) (2 points) Does $j(t)$ have any discontinuties? If so, what kind and where?

> **Solution:** Since this is a rational function, made up of nice functions, we know that discontinuities can happen only when $e^t = 0$, since that is the denominator. However, $e^t \neq 0$, so there are no discontinuities.

(e) (2 points) What are the horizontal asymptotes of $j(t)$?

> **Solution:** Before we are able to figure out the horizontal asymptotes, we must

compute the limits to infinity

$$\begin{aligned}\lim_{t\to\infty}\frac{t^3-t^2+1}{e^t} &= \frac{\infty}{\infty}\\ &\overset{\text{L'Hopital}}{=} \lim_{t\to\infty}\frac{3t^2-2t}{e^t}=\frac{\infty}{\infty}\\ &\overset{\text{L'Hopital}}{=} \lim_{t\to\infty}\frac{6t-2}{e^t}=\frac{\infty}{\infty}\\ &\overset{\text{L'Hopital}}{=} \lim_{t\to\infty}\frac{6}{e^t}=0.\end{aligned}$$

Similarly, we have

$$\lim_{t\to-\infty}\frac{t^3-t^2+1}{e^t}=-\infty$$

This means that the horizontal asymptote happen to be $y=0$ as $t\to\infty$.

- √ **Only** $j(t)=0$
- ☐ Only $j(t)=\infty$
- ☐ Both $j(t)=0$ and $j(t)=\infty$
- ☐ This function has no horizontal asymptotes.

(f) (2 points) Make a sign chart of the **first derivative**. What does it tell you about j?

Solution:
We proceed with the quotient rule:

$$\begin{aligned}j'(t)&=e^{-t}(3t^2-2t)-e^{-t}(t^3-t^2+1)\\ &=-e^{-t}\left(t^3-4t^2+2t+1\right)\\ &=-e^{-t}(t-1)(t^2-3t-1)\end{aligned}$$

The points where $j'(t)=0$ are when $t=1$ and when $t^2-3t-1=0$. This isn't factorable, so we will use the quadratic equation:

$$\begin{aligned}t&=\frac{3\pm\sqrt{(3)^2-4(1)(-1)}}{2(1)}\\ &=\frac{3\pm\sqrt{13}}{2}\end{aligned}$$

Lastly, we know that $-e^{-x}\neq 0$, so we have found all of the critical points. This means that the sign chart would look like:

j	$\nearrow$	$\rightarrow$	$\searrow$	$\rightarrow$	$\nearrow$	$\rightarrow$	$\searrow$
j'	$+$	0	$-$	0	$+$	0	$-$
x		$\frac{3}{2}-\frac{\sqrt{13}}{2}$		1		$\frac{3}{2}+\frac{\sqrt{13}}{2}$	

(g) (1 point) Find the point $(t, j(t))$ that is a *local minimum*.

Solution: Looking at the chart above, we see that $j(t)$ has two local minima when $j = 1$. This is since the derivative is decreasing before $t = 1$ and increasing after.

(h) (1 point) Find the point $(t, j(t))$ that is a *local maximum*.

Solution: We know that there are two local maximum: when $t = \frac{3}{2} - \frac{\sqrt{13}}{2}$ and $t = \frac{3}{2} + \frac{\sqrt{13}}{2}$. These are because the derivative is positive before them and negative after them. In fact, since the function decreases as $x \to \pm\infty$, then one of these points have to be a global max. We plug them in to the function to find that $t = \frac{3}{2} - \frac{\sqrt{13}}{2}$ is the global max.

(i) (3 points) Make a sign chart of the **second derivative**. What does it tell you about f? Are there any points of inflection?

Solution:
We proceed with the quotient rule:

$$\begin{aligned} j''(t) &= -e^{-t}(3t^2 - 8t + 2) + e^{-t}(t^3 - 4t^2 + 2t - 1) \\ &= e^{-t}\left(t^3 - 7t^2 + 10t - 1\right) \end{aligned}$$

This exists for all points t. This has roots where $t \approx 0.108, 1.828, 5.064$.

j	$\cap$		$\cup$		$\cap$		$\cup$
j''	$-$	0	$+$	0	$-$	0	$+$
t		0.108		1.828		5.064	

We see that all of these points are inflection points.

(j) (10 points) Using the information from the previous parts, sketch the graph of the function:

$$j(t) = \frac{t^3 - t^2 + 1}{e^t}$$

Solution: