Emory University
Exam # 3 Review Sheet
MA-111 Calculus I
Date: November 14, 2025
Instructor: Prof. Mitchell Scott

Student ID _____ Name: _____

Please read the following instructions carefully.

- You have 90 minutes to complete this exam. This question booklet contains 4 questions, 10 pages (including the cover) for the total of 108 points/marks. Check to see if any pages are missing. DO NOT scribble or do rough work or make any stray marks on it. Use separate sheet for rough work.
- All the questions are compulsory and all the notations have their usual meaning. You can use standard results directly by writing its statement appropriately. **Read the instructions for individual questions carefully** before answering the questions.
- More instructions here.

Question	Points	Score
1	27	
2	27	
3	27	
4	27	
Total:	108	

1. This problem will guide you through the steps to sketch the graph of the following function:

$$f(x) = \frac{e^x}{(x+2)^2.}$$ ⇒ rational

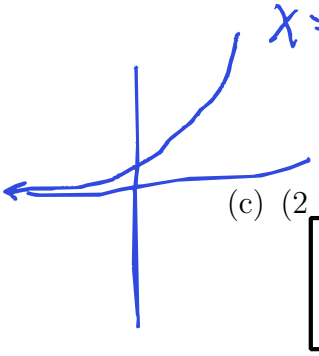
(a) (2 points) What is the domain of $f(x)$?

$(x+2)^2 = 0 \Rightarrow x+2 = 0 \Rightarrow x = -2$

Domain $f(x)$ = all real numbers EXCEPT $x = -2$

(b) (2 points) Find the y-intercept of $f(x)$

$x = 0 \quad f(0) = \frac{e^0}{(0+2)^2} = \frac{1}{4}$



(c) (2 points) Find the x-intercept of $f(x)$, if there is one.

$e^x \neq 0$ There are no x values that satisfy this equation

$x = -2$

(d) (2 points) Does $f(x)$ have any discontinuities? If so, what kind and where?

$\lim_{x \to -2^-} \frac{e^x}{(x+2)^2} = \frac{+}{(0^-)^2} = \frac{+}{+} \to \infty \qquad \lim_{x \to -2^+} \frac{e^x}{(x+2)^2} = \frac{+}{(0^+)^2} \to \infty$

VA

(e) (2 points) What are the horizontal asymptotes of $f(x)$?

Workspace:

HA limits to infinity

$\lim_{x \to \infty} \frac{e^x}{(x+2)^2} \overset{LH}{=} \lim_{x \to \infty} \frac{e^x}{2(x+2)} \overset{LH}{=} \lim_{x \to \infty} \frac{e^x}{2} = \infty$

$e^{-x} = \frac{1}{e^{+x}}$

$\lim_{x \to -\infty} \frac{e^x}{(x+2)^2} = \frac{e^{-\infty}}{\infty} = \frac{1}{e^{\infty} \cdot \infty} = 0$

- [x] Only $f(x) = 0$
- [ ] Only $f(x) = \infty$
- [ ] Both $f(x) = 0$ and $f(x) = \infty$
- [ ] This function has no horizontal asymptotes.

$f(x) = \frac{e^x}{(x+2)^2}$

(f) (2 points) Make a sign chart of the **first derivative**. What does it tell you about f?

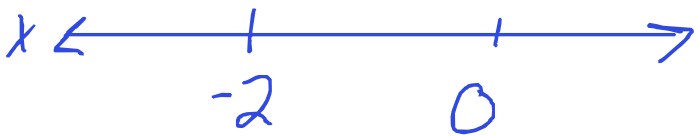
$f'(x) = \frac{(x+2)^2 e^x - e^x(2(x+2))}{((x+2)^2)^2} = \frac{(x+2)e^x - 2e^x}{(x+2)^3} = \frac{xe^x}{(x+2)^3}$

$$f'(x) = \frac{xe^x}{(x+2)^3}$$

Crit points

$f'(x) = 0$ or DNE

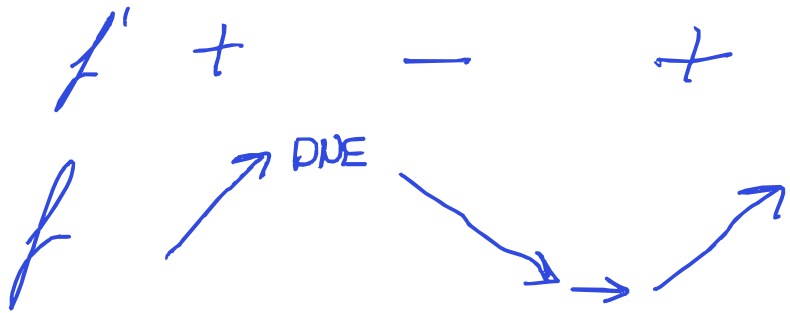
$x = -2 \quad x = 0$



$$f'(-5) = \frac{-5e^{-5}}{(-3)^3}$$

$$f'(-1) = \frac{-1e^{-1}}{(1)^3}$$

$$f'(1) = \frac{e}{(3)^3}$$



(g) (1 point) Find the point $(x, f(x))$ that is a *local minimum*.

$x^2+2=0$
$x^2=-2$

$x=0$ so $\frac{e^0}{2^2}=\frac{1}{4}$

(h) (1 point) Find the point $(x, f(x))$ that is a *local maximum*.

$f''(x)$ DNE at $x=-2$

No local max

$f''(x)=0$

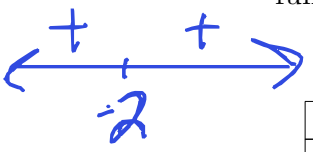
$f'(x)=\frac{xe^x}{(x+2)^3}$

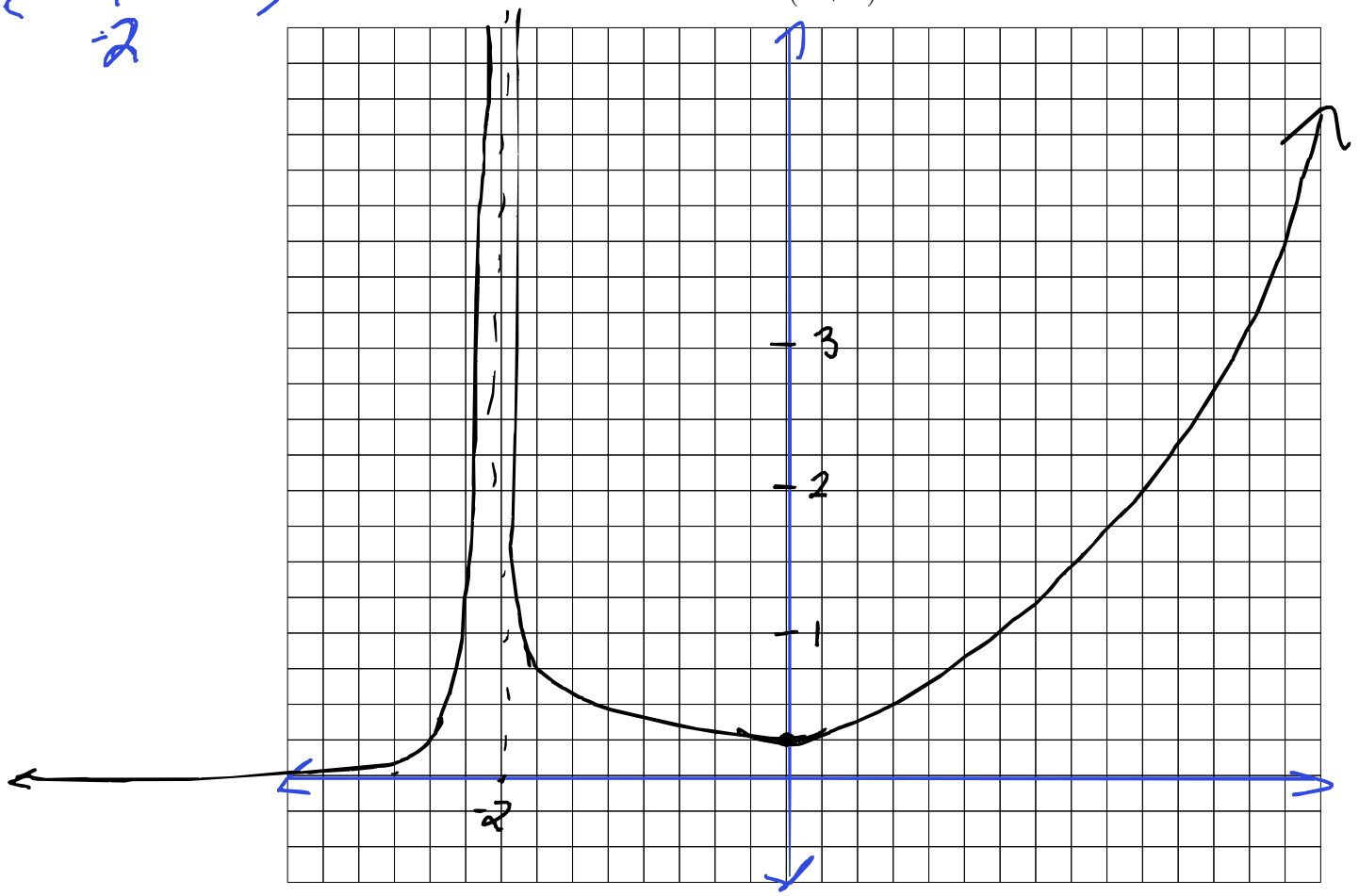
(i) (3 points) Make a sign chart of the **second derivative**. What does it tell you about f? Are there any points of inflection?

$f''(x)=\frac{(x+2)^3(xe^x+e^x)-xe^x(3(x+2)^2)}{(x+2)^6}=\frac{e^x(x^2+2)}{(x+2)^4}$

(j) (10 points) Using the information from the previous parts, sketch the graph of the function:

$$f(x)=\frac{e^x}{(x+2)^2}$$





2. This problem will guide you through the steps to sketch the graph of the following function:

$$g(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + 3$$

(a) (2 points) What is the domain of $g(x)$?

(b) (2 points) Find the y-intercept of $g(x)$

(c) (2 points) Find the x-intercept of $g(x)$, if there is one.

(d) (2 points) Does $g(x)$ have any discontinuties? If so, what kind and where?

(e) (2 points) What are the horizontal asymptotes of $g(x)$?
Workspace:

- ☐ Only $g(x) = 0$
- ☐ Only $g(x) = \infty$
- ☐ Both $g(x) = 0$ and $g(x) = \infty$
- ☐ This function has no horizontal asymptotes.

(f) (2 points) Make a sign chart of the **first derivative**. What does it tell you about g?

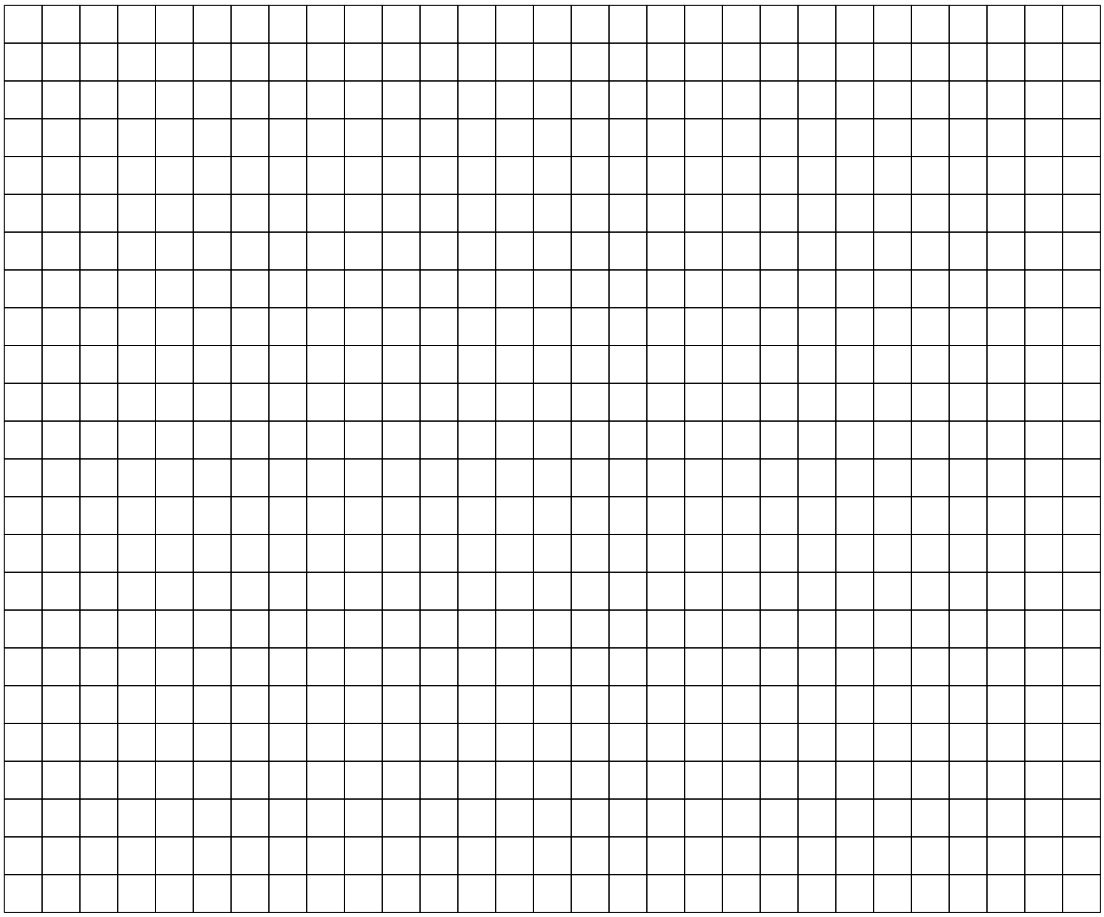
(g) (1 point) Find the point $(x, g(x))$ that is a *local minimum*.

(h) (1 point) Find the point $(x, g(x))$ that is a *local maximum*.

(i) (3 points) Make a sign chart of the **second derivative**. What does it tell you about g? Are there any points of inflection?

(j) (10 points) Using the information from the previous parts, sketch the graph of the function:

$$g(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + 3$$



3. This problem will guide you through the steps to sketch the graph of the following function:

$$h(x) = \frac{x^2 - x - 2}{x^2 - x - 6}.$$

Rational function

(a) (2 points) What is the domain of $h(x)$?

$(x^2-x-6) = (x-3)(x+2) = 0$ so

all real numbers except $x = -2, 3$

(b) (2 points) Find the y-intercept of $h(x)$

$$h(0) = \frac{0^2-0-2}{0^2-0-6} = \frac{-2}{-6} = \frac{1}{3}$$

(c) (2 points) Find the x-intercept of $h(x)$, if there is one.

$h(x) = 0$ so $x^2-x-2 = (x-2)(x+1) = 0$ so

$h(2) = h(-1) = 0$

(d) (2 points) Does $h(x)$ have any discontinuties? If so, what kind and where?

$x = -2$
$x = 3$

$$\lim_{x \to -2^+} \frac{(-2)^2-(-2)-2}{(-2-3)(0^+)} = \frac{4}{0^+} = \frac{4}{0^+} = +\infty$$

$x \to -2^- = -\infty$

(e) (2 points) What are the horizontal asymptotes of $h(x)$?

$\lim_{x \to 3^-} -\infty$ $\lim_{x \to 3^+} +\infty$

Workspace:

$$\lim_{x \to \infty} \frac{x^2-x-2}{x^2-x-6} \overset{LH}{=} \lim_{x \to \infty} \frac{2x-1}{2x-1} \overset{LH}{=} \frac{2}{2} = 1$$

$\lim_{x \to -\infty} h(x) = 1$

- ☐ Only $h(x) = \infty$
- ■ Only $h(x) = 1$
- ☐ Both $h(x) = 0$ and $h(x) = \infty$
- ☐ This function has no horizontal asymptotes.

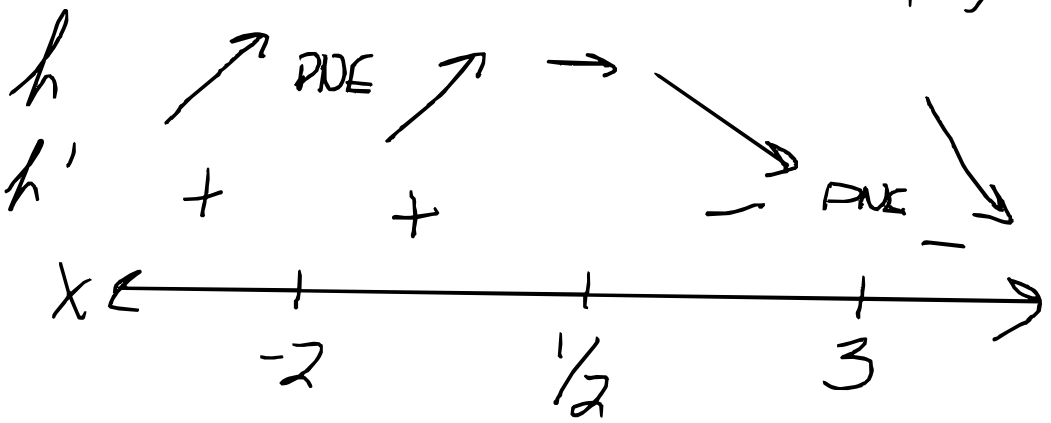
(f) (2 points) Make a sign chart of the **first derivative**. What does it tell you about h?

$$h'(x) = \frac{(x^2-x-6)(2x-1) - (x^2-x-2)(2x-1)}{(x^2-x-6)^2} = \frac{-8x+4}{(x-3)^2(x+2)^2}$$

$$h'(x) = \frac{-8x+4}{(x-3)^2(x+2)^2}$$

$h'(-5) = \frac{+}{+}$ $h'(1) = \frac{-}{+}$

$h'(0) = \frac{+}{+}$ $h'(5) = \frac{-}{+}$



$x = 1/2$ is local maximum

h ∪ DNE ∩ DNE ∪
h'' + DNE − DNE +
-2 3

(g) (1 point) Find the point $(x, h(x))$ that is a *local minimum*.

(h) (1 point) Find the point $(x, h(x))$ that is a *local maximum*.

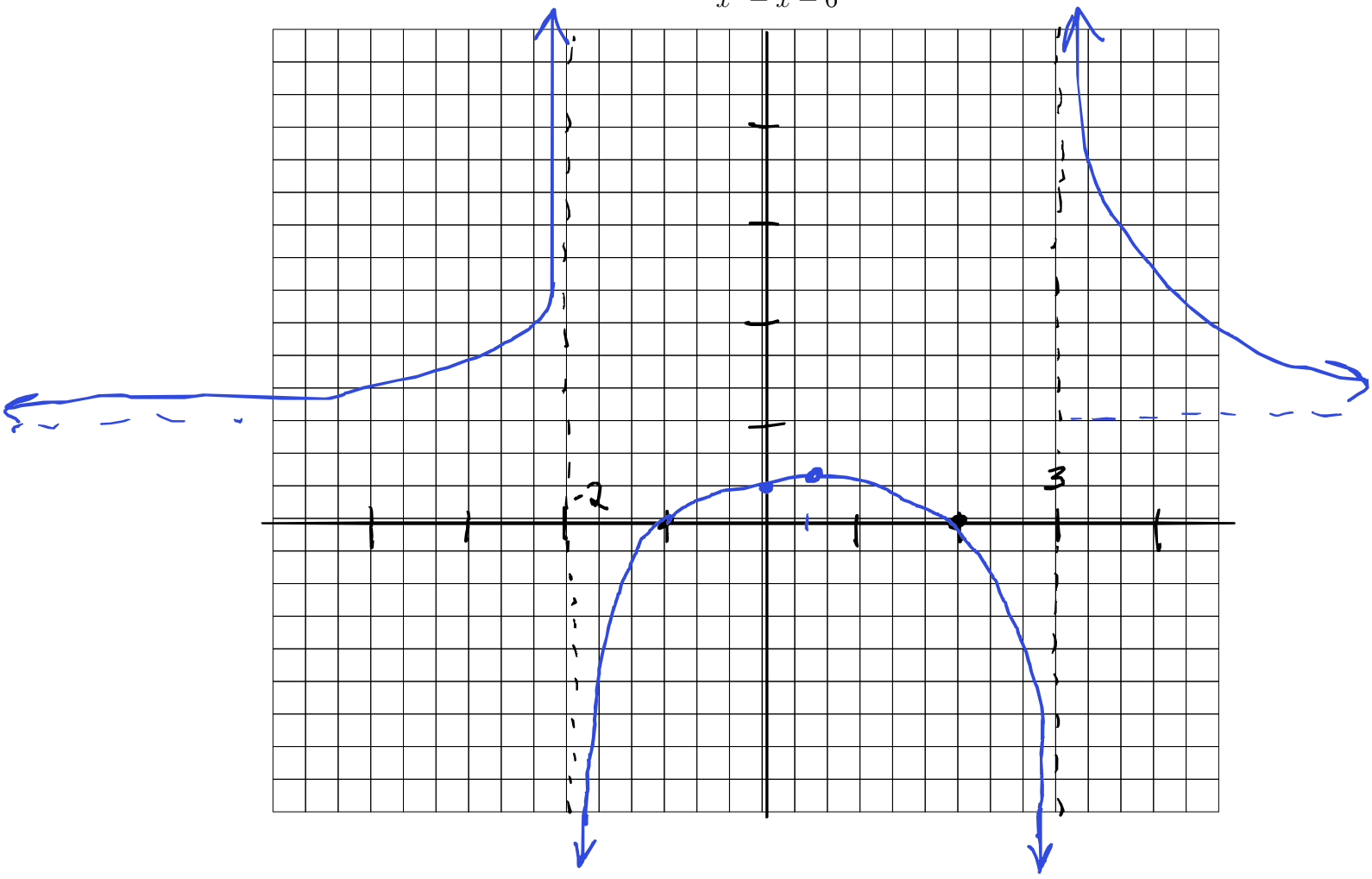
$(x-3)(x+2)$ $\sqrt{3^2-4\cdot 3\cdot 7}$

(i) (3 points) Make a sign chart of the **second derivative**. What does it tell you about h? Are there any points of inflection?

$$h''(x) = \frac{(x-3)^2(x+2)^2(8) - (-8x+4)(2)(x^2-x-6)(2x-1)}{(x-3)^4(x+2)^4} = \frac{24x^2-24x+56}{(x-3)^3(x+2)^3}$$

(j) (10 points) Using the information from the previous parts, sketch the graph of the function:

$$h(x) = \frac{x^2-x-2}{x^2-x-6}$$



4. This problem will guide you through the steps to sketch the graph of the following function:

$$j(t) = \frac{t^3 - t^2 + 1}{e^t}$$

(a) (2 points) What is the domain of $j(t)$?

(b) (2 points) Find the y-intercept of $j(t)$

(c) (2 points) Find the t-intercept of $j(t)$, if there is one.

(d) (2 points) Does $j(t)$ have any discontinuties? If so, what kind and where?

(e) (2 points) What are the horizontal asymptotes of $j(t)$?
Workspace:

- ☐ Only $j(t) = 0$
- ☐ Only $j(t) = \infty$
- ☐ Both $j(t) = 0$ and $j(t) = \infty$
- ☐ This function has no horizontal asymptotes.

(f) (2 points) Make a sign chart of the **first derivative**. What does it tell you about j?

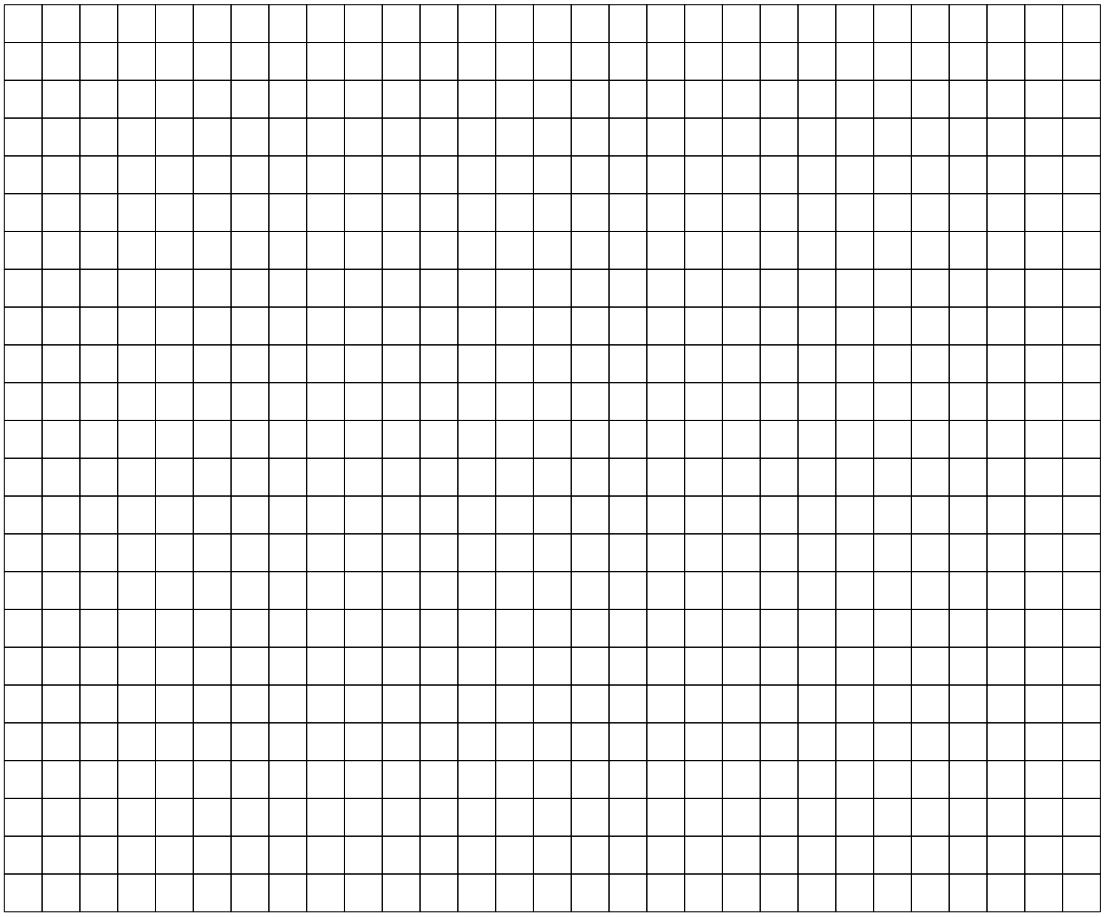
(g) (1 point) Find the point $(t, j(t))$ that is a *local minimum*.

(h) (1 point) Find the point $(t, j(t))$ that is a *local maximum*.

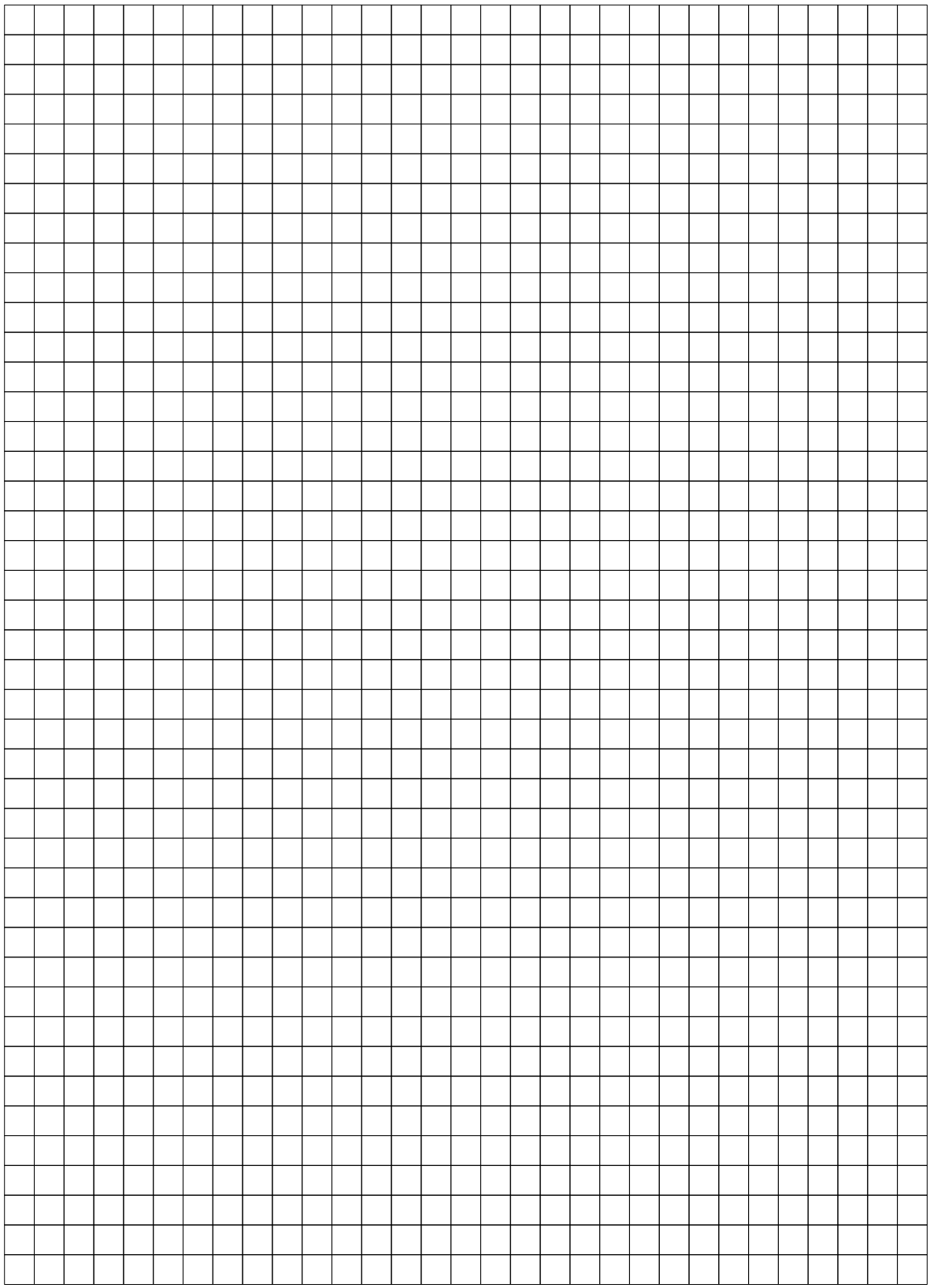
(i) (3 points) Make a sign chart of the **second derivative**. What does it tell you about f? Are there any points of inflection?

(j) (10 points) Using the information from the previous parts, sketch the graph of the function:

$$j(t) = \frac{t^3 - t^2 + 1}{e^t}$$



Extra grid paper:



End of exam.